

REFINED GENETIC ALGORITHMS
FOR
POLYPEPTIDE STRUCTURE PREDICTION

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REFINED GENETIC ALGORITHMS
FOR
POLYPEPTIDE STRUCTURE PREDICTION

by [Name] and [Name]

Department of [Department Name]

[Institution Name]

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Acknowledgements

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α
\mathcal{F}
\mathcal{S}
α
γ
i	i
ϕ
ψ
ω
χ_i	i

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Abstract

I. Introduction

predict

*native conformation*¹

$$PSP \subseteq PFP$$

¹The *native conformation* determines the protein's biological functions.

1.1.3 Methods for Polypeptide Structures Prediction.

energy minimization molecular dynamics

ab initio

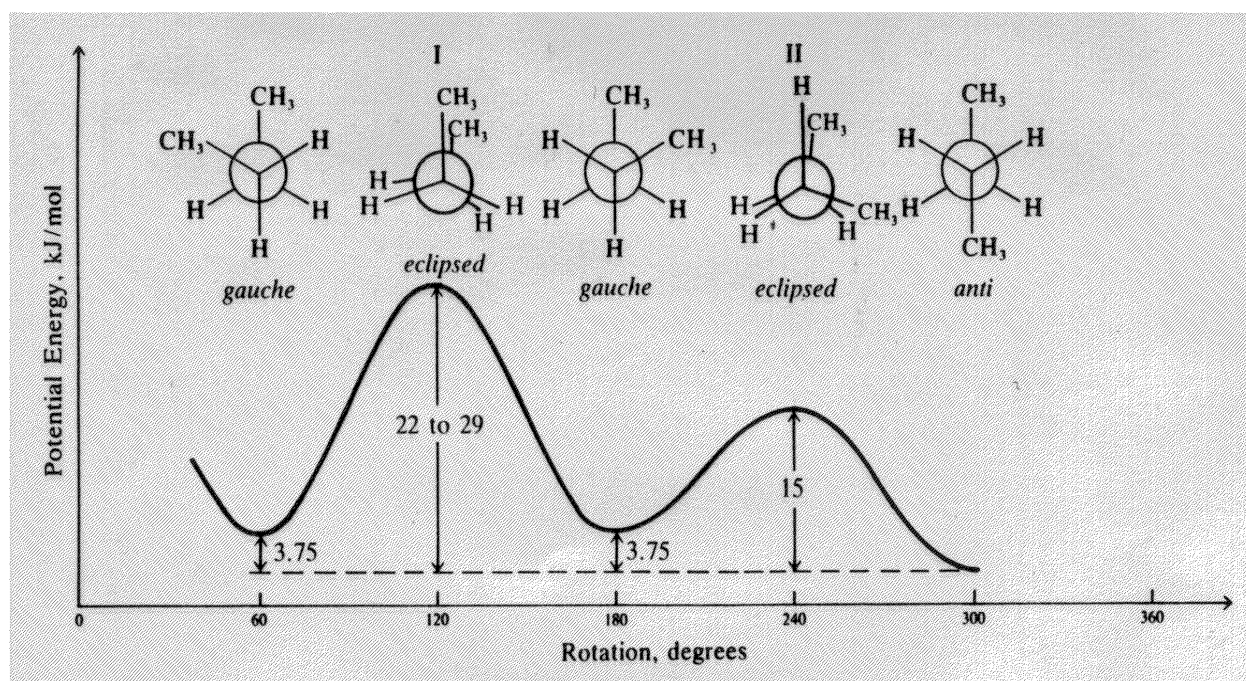
semi-empirical

force-field

$\mathcal{O} n^5$ $\mathcal{O} n^4$

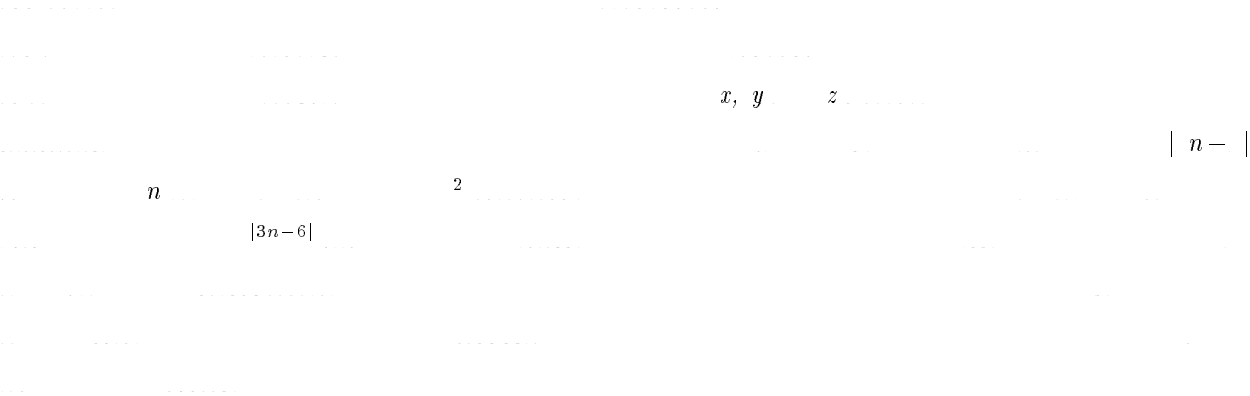
$\mathcal{O} n^2$

n

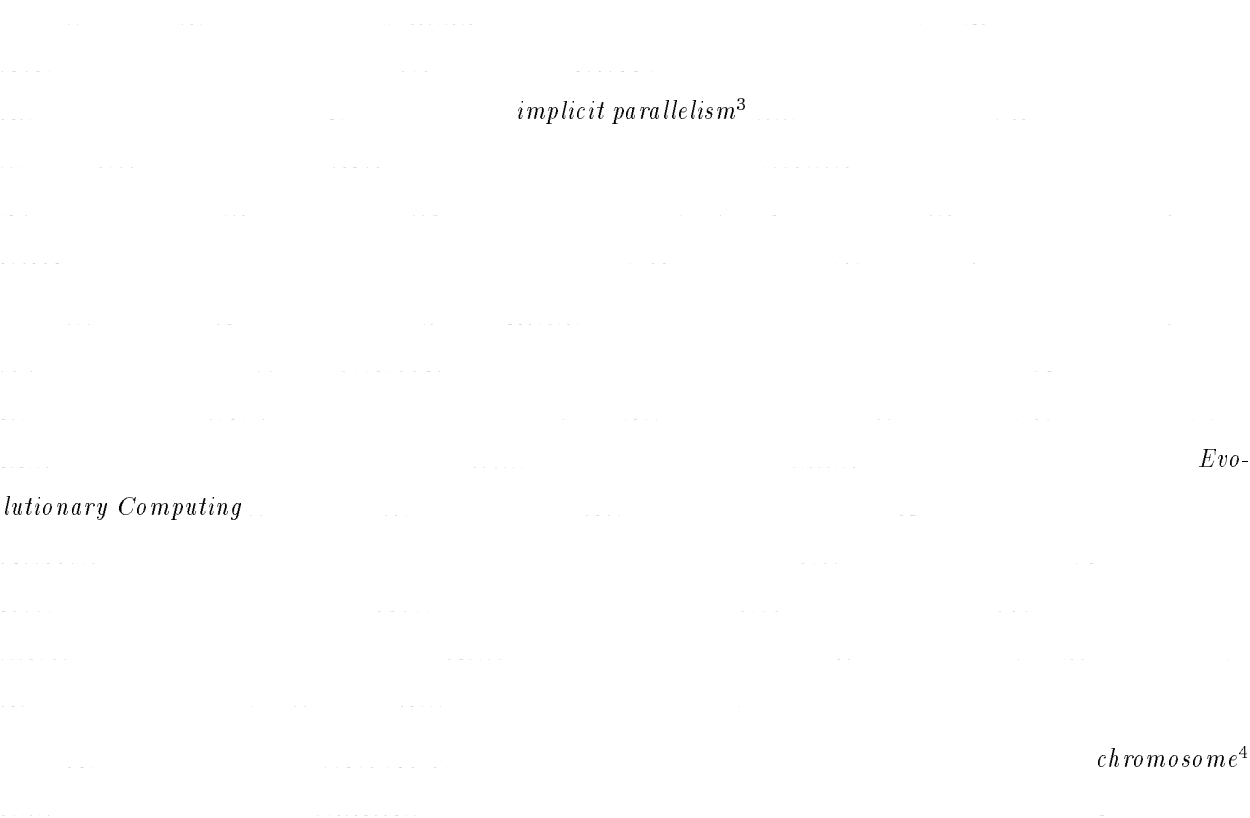


n

1.1.4 Growth of Complexity.

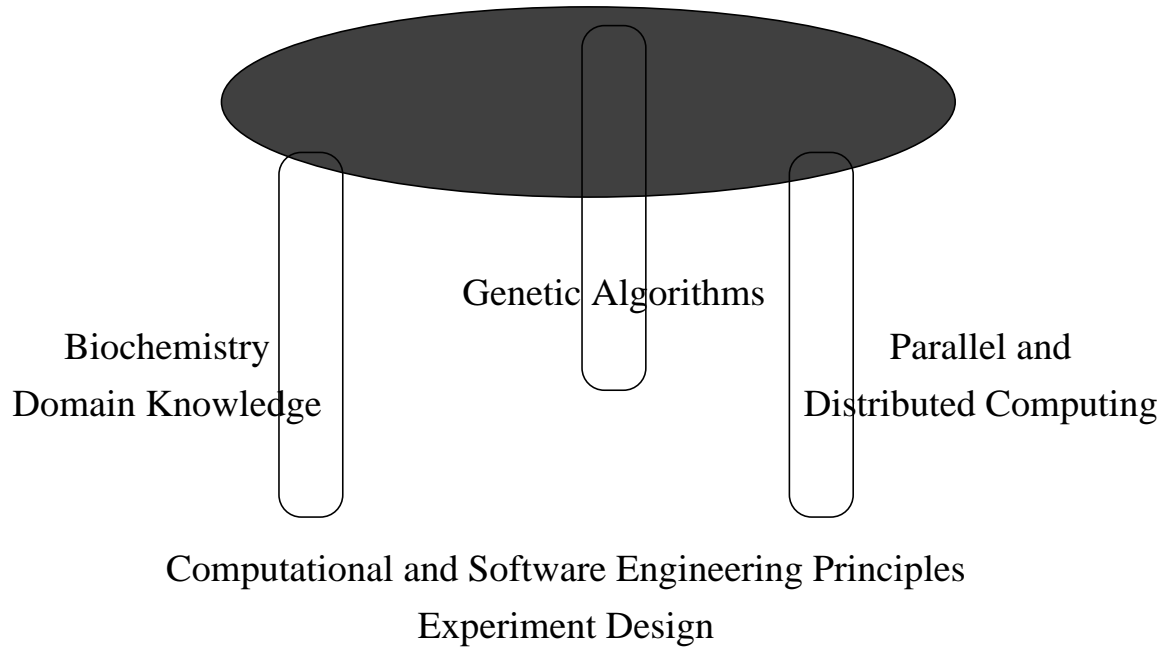


1.2 Genetic Algorithms



²Regardless of whether a *Cartesian* or *Internal* coordinate system is used. However, the internal coordinate system has fewer independent variables
³See Appendix B.3.3
⁴Because GAs are loosely based on natural evolution, many of the terms associated with natural evolution are used interchangeably with the terms created specifically for genetic algorithms (67).

Effective and Efficient
Polypeptide Structure Prediction



- *Improve Performance of Hybrid GAs for PSP*
- *Real Valued Genetic Algorithm Implementation for the PSP*
- *Exploit Domain Knowledge to Limit Search Space*
domain knowledge

1.5 Methodology

1.6 Assumptions

1.7 Summary

⁶Not to be confused with *Evolutionary Programming*, see Section 2.4.

⁷The probability that an improvement at a specific node is migrated to other nodes.

⁸The probability, given a migration, that it is migrated to all other nodes.

⁹ Or molecular conformation

II. Current Issues

2.1 Introduction

Polypeptides are linear chains of amino acids, which are the building blocks of proteins. The sequence of amino acids in a polypeptide chain determines its structure and function. The structure of a polypeptide chain is determined by the sequence of amino acids, which is encoded in the DNA sequence. The function of a polypeptide chain is determined by its structure, which is determined by the sequence of amino acids. The structure of a polypeptide chain is determined by the sequence of amino acids, which is encoded in the DNA sequence. The function of a polypeptide chain is determined by its structure, which is determined by the sequence of amino acids.

2.2 Previous Research

Previous research has shown that the structure of a polypeptide chain is determined by the sequence of amino acids. The function of a polypeptide chain is determined by its structure, which is determined by the sequence of amino acids. The structure of a polypeptide chain is determined by the sequence of amino acids, which is encoded in the DNA sequence. The function of a polypeptide chain is determined by its structure, which is determined by the sequence of amino acids.

2.3 Polypeptides Structure Prediction (PSP)

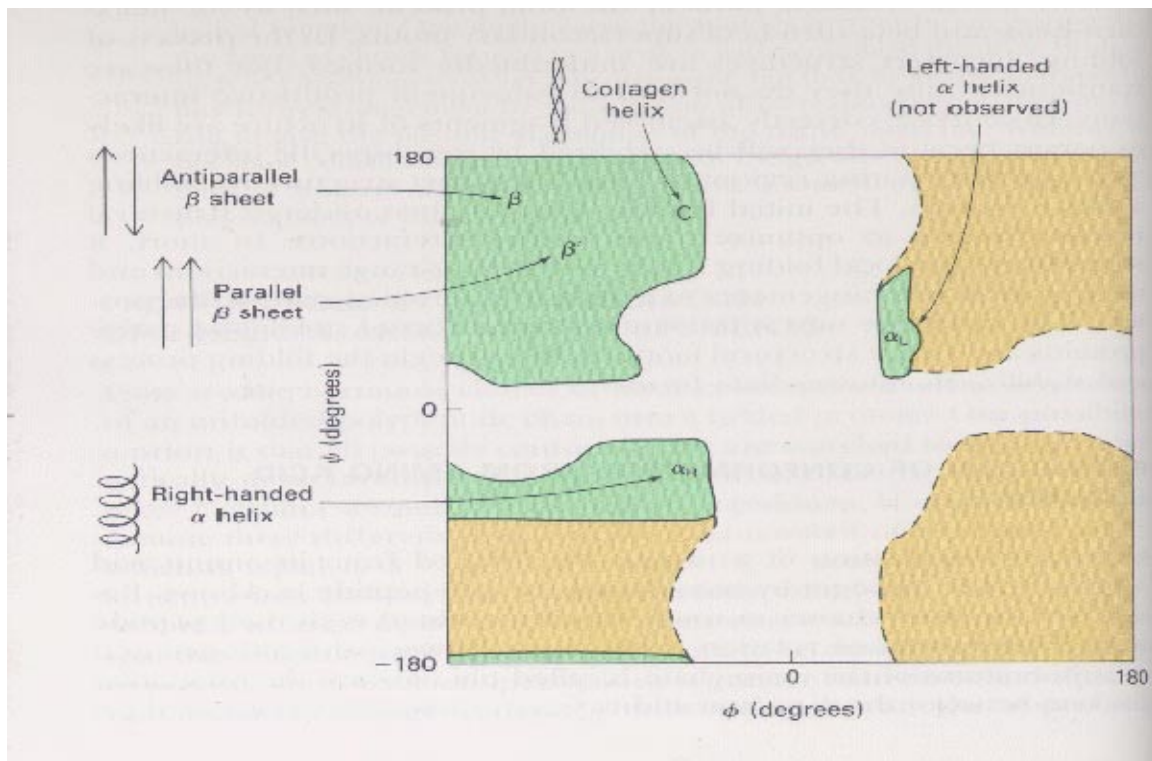
Polypeptides Structure Prediction (PSP) is a field of research that aims to predict the structure of a polypeptide chain from its sequence. The structure of a polypeptide chain is determined by the sequence of amino acids, which is encoded in the DNA sequence. The function of a polypeptide chain is determined by its structure, which is determined by the sequence of amino acids. The structure of a polypeptide chain is determined by the sequence of amino acids, which is encoded in the DNA sequence. The function of a polypeptide chain is determined by its structure, which is determined by the sequence of amino acids.



Ramachandran Plot

ψ

ϕ



Biochemistry

2.4 Genetic Algorithms

evolution

program

paradox of real codings

virtual alphabets

blocked

Fundamental Theorem of Genetic Algorithms

strong *weak*

 EP_1 EP_5 EP_1 EP_5
$$P$$

data parallelism

island model

neighborhood model . . .

farming model

Island Model:

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..... *migration*
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..... $\mathcal{O} \frac{nl}{p}$ p n
..... $p \ll n$ $p \rightarrow n$
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Neighborhood Model:

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..... n p
.....
..... $\mathcal{O} s \cdot l$ $\mathcal{O} nl$ s
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Farming Model: *farm*
..... *farm out*
..... *foreman* *workers*
.....
.....
.....

2.6 Summary

III. Algorithm Analysis, Design, and Implementation

3.1 Analysis

3.1.1 Cost Analysis of Local Minimization using Conjugate Gradient.

Numerical Recipes in C

n

n

$\mathcal{O}(n^2)$

ITMAX = 200

Lamarckian *Baldwinian*

eval_func()

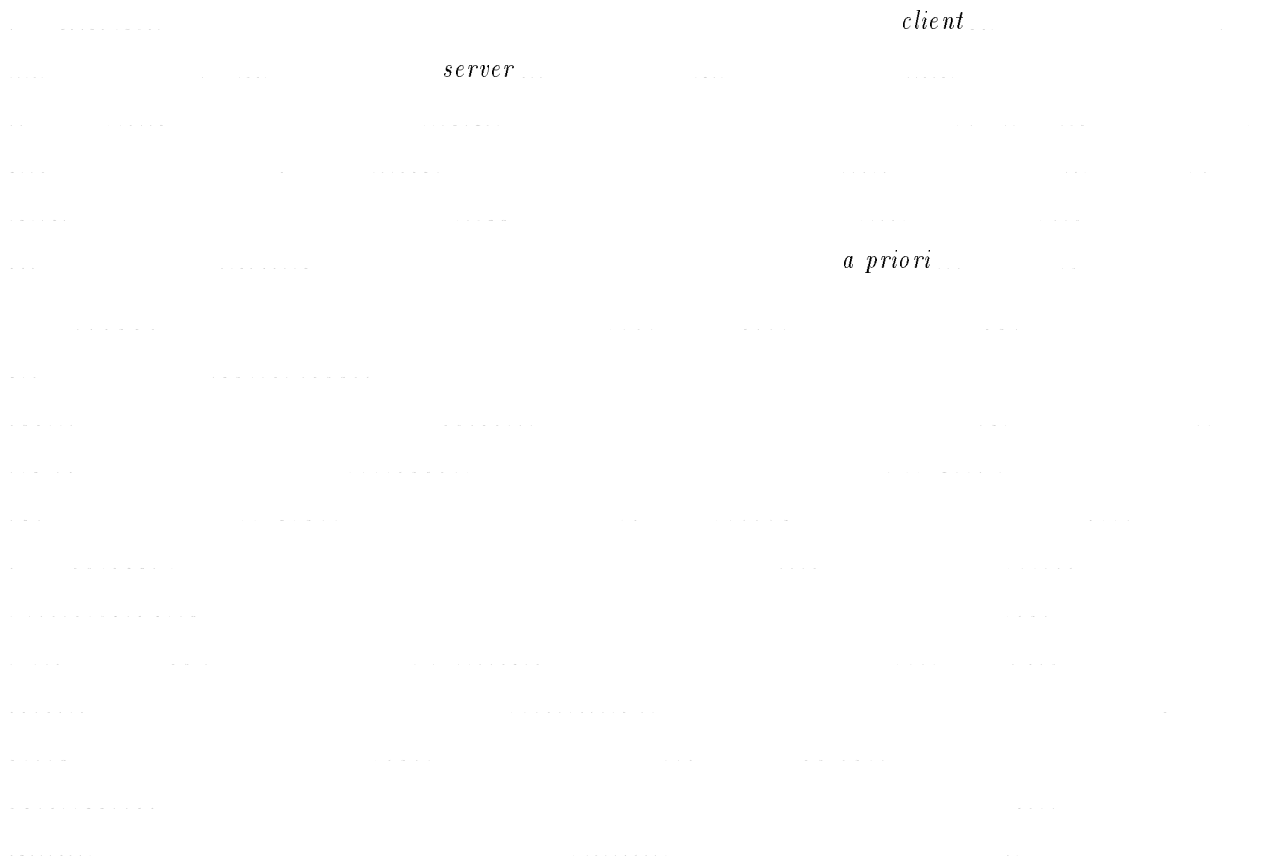
ITMAX

generational

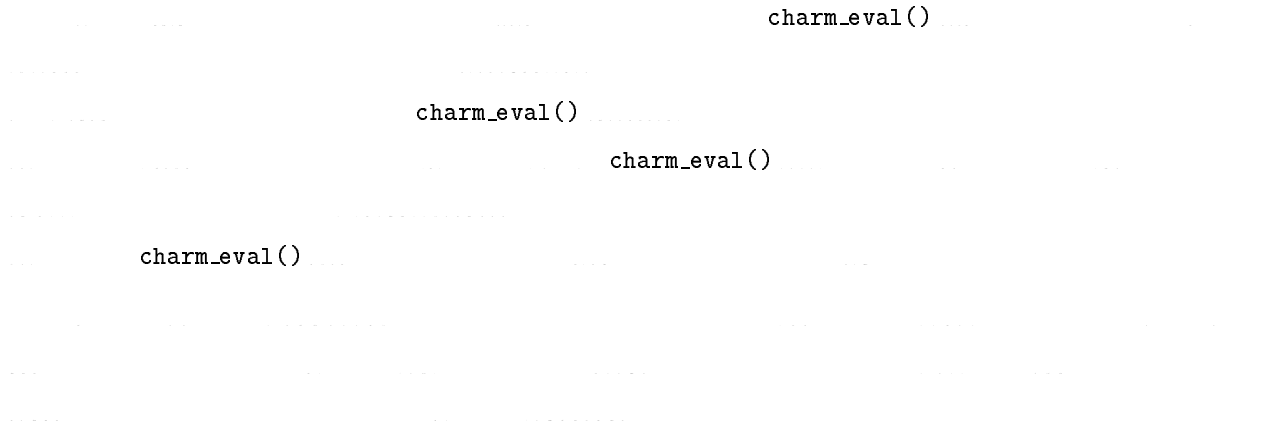
checkpoint

asynchronous

farming

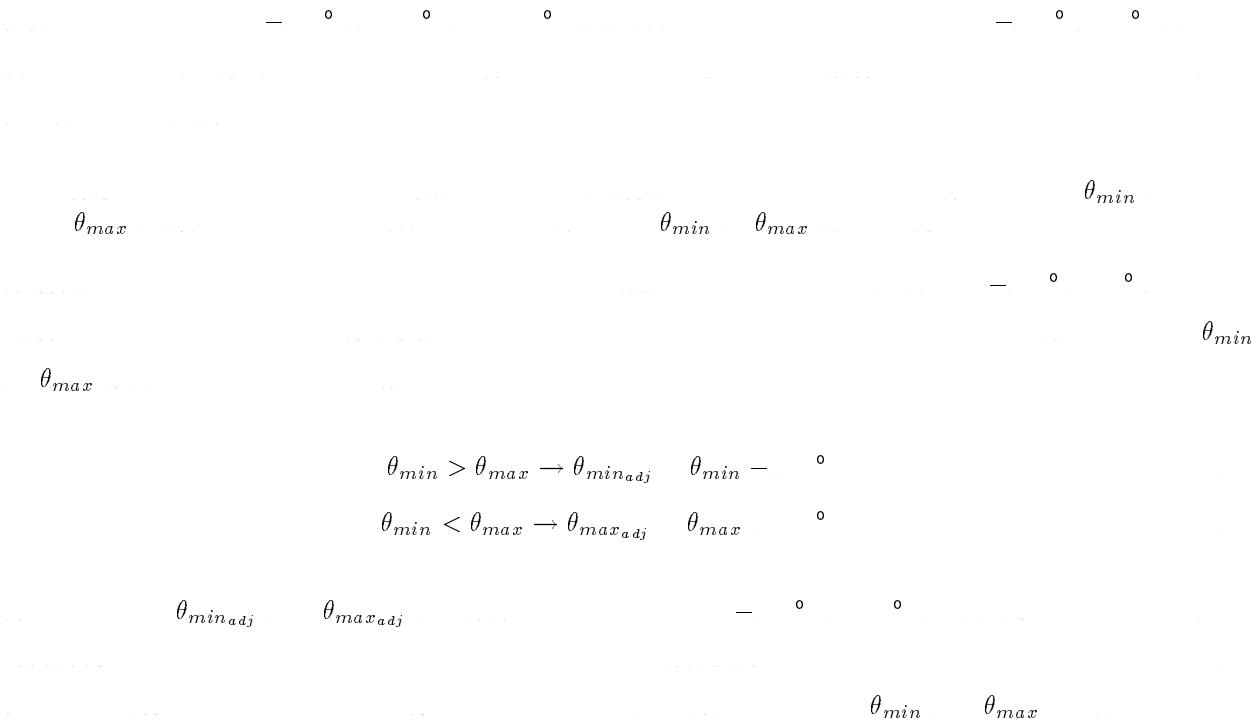


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3.1.2 Constraint Set Development.

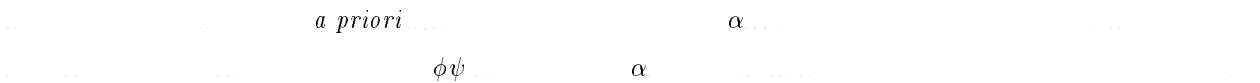
3.1.2.1 Conventions Adopted.



3.1.2.2 [Met]-enkephalin.



3.1.2.3 Polyalanine.



				θ_{min}	θ_{max}
	—			—	—
	—			—	—
				—	—
χ_1	—			—	—

3.2 Algorithm Design and Implementation

3.2.1 Parallel Hybrid GA.

farming model

3.2.1.1 Algorithm Design.

G_{t+1} G_t

¹The developers of this algorithm have been inconsistent with its naming in the literature. Genocop, Genocop-III.1.0, GENOCOP-III, etc., have been used synonymously. In this document I have adopted the standardization of GENOCOP-III.

```

-----
-----
       $Q$ 
EvalCnt ← population size
Loop
  Until EvalCnt ...
  Loop ...
  ...
  ...
  ...
  EvalCnt ← ...
  Loop
    ...
    Until EvalCnt ...
  Until ...
-----
-----

```

3.2.1.2 Scheduling.

round robin

network of workstations

3.2.2 REal-valved GA, Limited by constraints (REGAL) .

evolution program

Loop

Until ...

REPLACE

$server_i \quad Q$
 $EvalsPerformed$
 $EvalsPerformed < EvalCnt \wedge Q \neq \emptyset$
 $server_j \quad Q$

server_j

3.2.2.1 Incorporation of Domain Knowledge.

Evolution Program

probable *improbable* \mathcal{S}_{prob}

\mathcal{S}_{improb}

$\mathcal{S}_{prob} \quad \mathcal{S}_{improb} \quad \mathcal{S}$

$\mathcal{S}_{prob} \quad \mathcal{S}_{improb} \quad \emptyset$

$$x_i \in \left[\frac{\theta_{i_{min}} + \theta_{i_{max}}}{2} - \frac{\theta_{i_{max}} - \theta_{i_{min}}}{2} \cos \left(\frac{\pi}{2} \right), \frac{\theta_{i_{min}} + \theta_{i_{max}}}{2} + \frac{\theta_{i_{max}} - \theta_{i_{min}}}{2} \cos \left(\frac{\pi}{2} \right) \right]$$

$$\leq \left[\theta - \frac{\theta_{min} - \theta_{max}}{2} - \frac{\theta_{max} - \theta_{min}}{2} \right]$$

$$\{\phi, \psi, \omega\}$$

$$\leq \left[\theta - \frac{\theta_{min} - \theta_{max}}{2} - \frac{\theta_{max} - \theta_{min}}{2} \right]$$

$$\{\chi_1\}$$

3.2.3 Parallel REGAL (Para-REGAL).

modified island

island

²

probabilistic migration

Probability of Migration

P_m *Probability of Complete Migration* P_{cm}

$P_m - P_{cm}$

i

j

$P_m \cdot P_{cm}$

$P_m \cdot P_{cm} >$

*archipelago*³

- *Average Genotypic Distance*

- *Least Genotypic Distance*

²Except for the seed of the random number generator.

³A collection of islands; thus the islands making of the Para-REGAL execution

4.1.2.2 Kruskal-Wallis H Test.

4.2 Experiment I: Evaluation of the Efficiency of a Parallel & Distributed Hybrid GA

4.2.1 Motivation and Objective.

4.2.2 Methodology.

$$P_r = \frac{P_m}{P_{max}}$$

farming model

.....

.....

			P_m	P_r

.....

..... 987654321

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4.3 Experiment II: Evaluation of the Use of Constraints in the PSP

4.3.1 Motivation and Objective.

.....

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4.3.2 Methodology.

.....

..... none loose tight

[Met]-enkephalin tight Polyalanine tight

4.3.3 Parameter Selection.

.....

..... reference population

..... \mathcal{F} \mathcal{S}

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loose tight

4.4 Experiment III: Evaluation of Exogenous Parameters in the REGAL System

4.4.1 Motivation and Objective.

¹Based on the ratio of $\frac{\mathcal{F}}{\mathcal{S}}$, it would require 10^{67} tries to randomly generate just on fully feasible chromosome when using the *tight* constraint set for Polyalanine.

4.4.3	<i>Exogenous Parameter Evaluation Experiments.</i>
	<i>Offsprings</i>	<i>Reference Population Size</i>

4.5 Experiment IV: Evaluation of Para-REGAL

4.5.1 Motivation and Objective.

<i>Island 0</i>			
<i>Islands 1 & 2</i>	<i>loose</i>	<i>Island 3</i>	<i>tight</i>

4.5.3 Para-REGAL Experiments.

$$\{ \cdot , \cdot , \cdot , \cdot , \cdot , \cdot \} \quad P_m = P_{em} =$$

		ω		

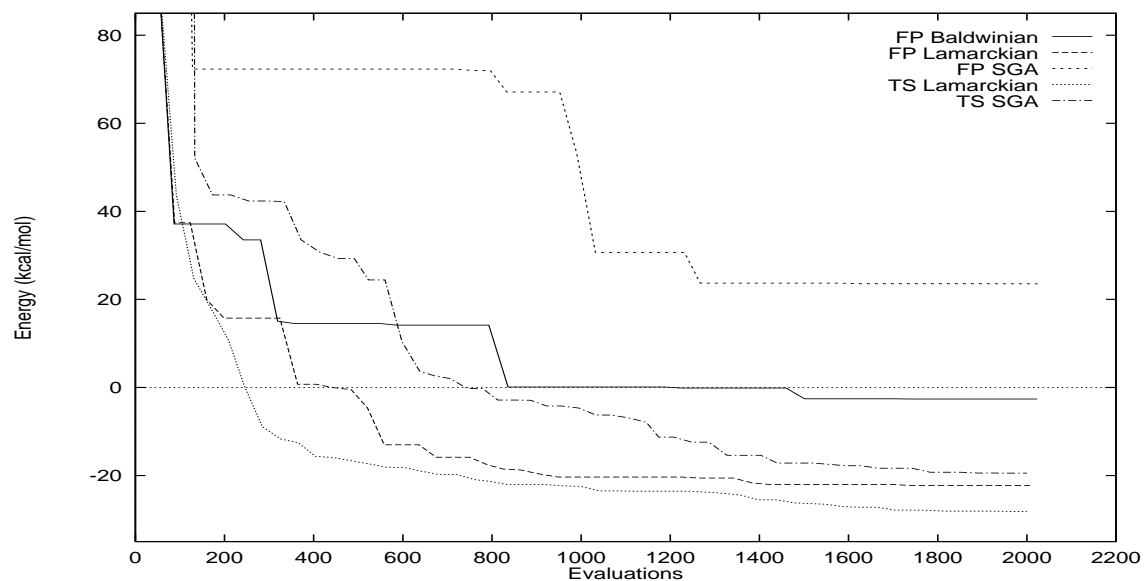
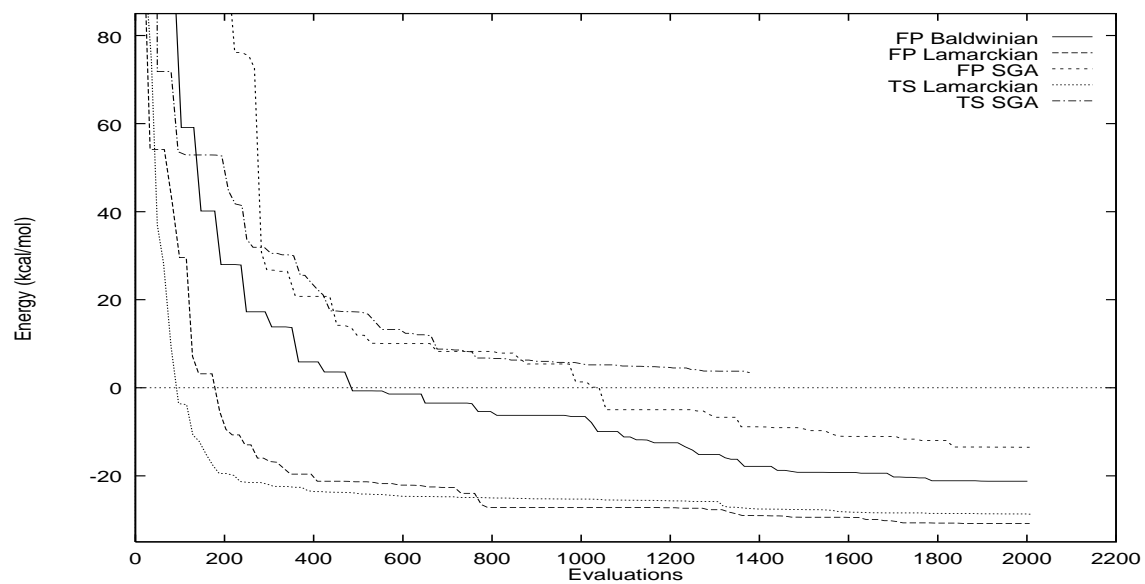
V. Results and Analysis

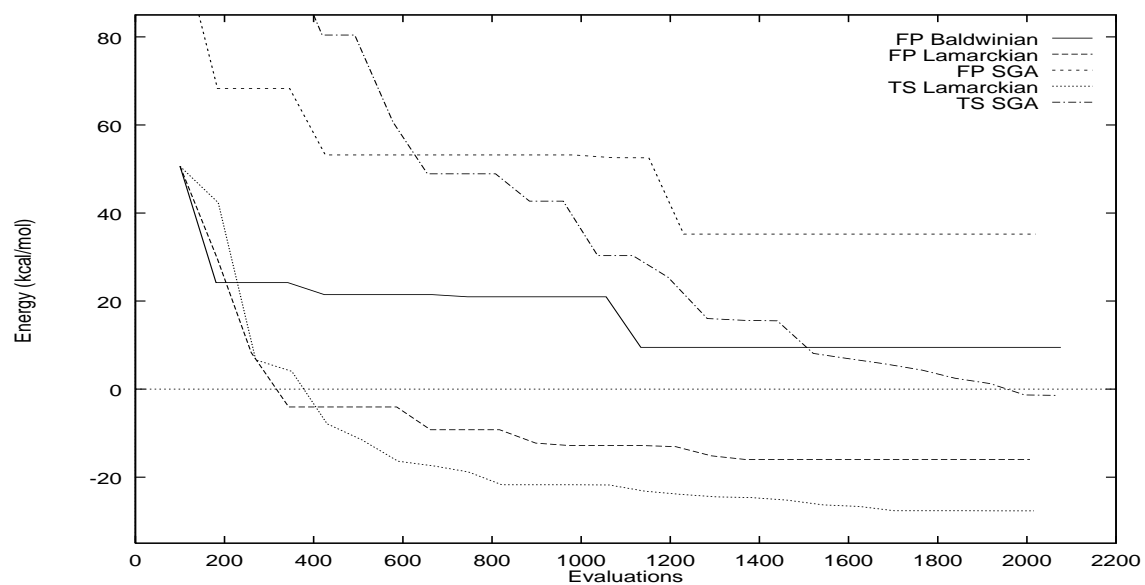
5.1 Experiment I: Parallel Hybrid GA

5.1.1 Effectiveness Analysis.

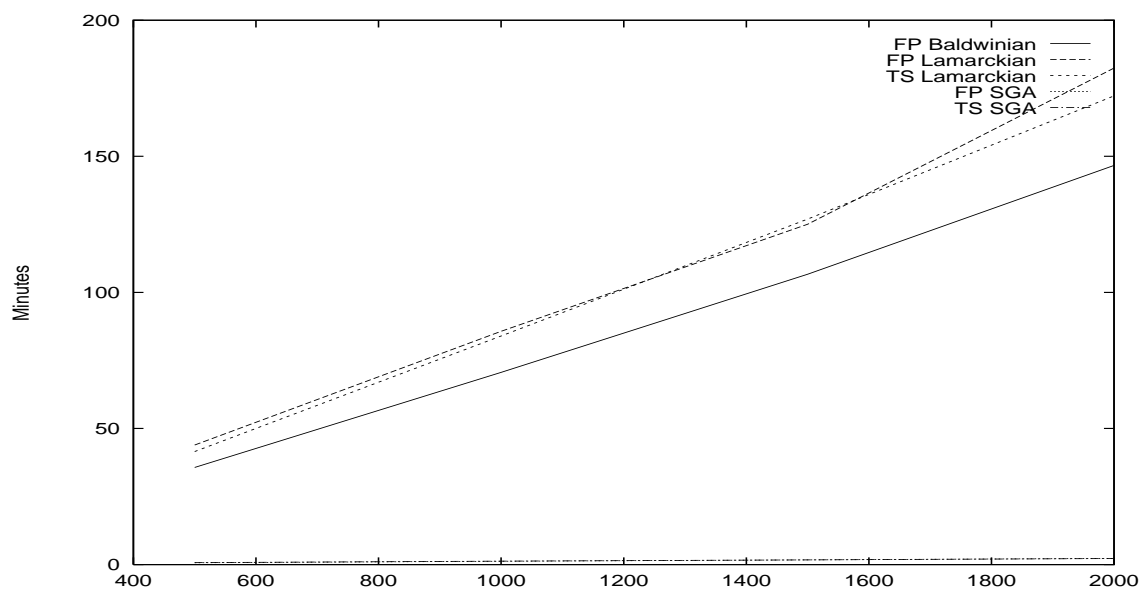
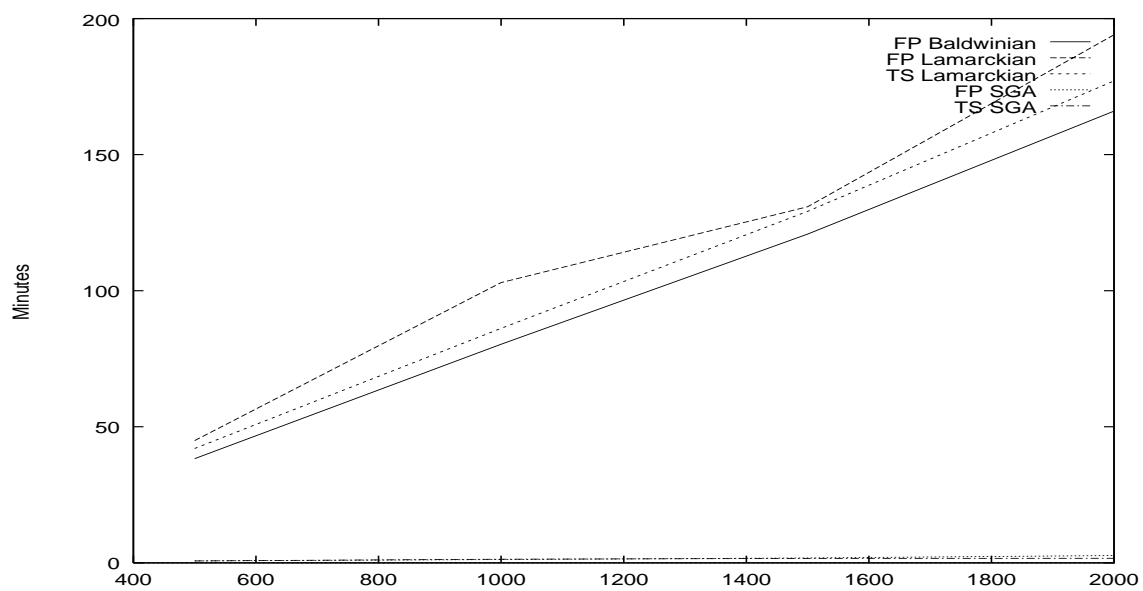
selection pressure

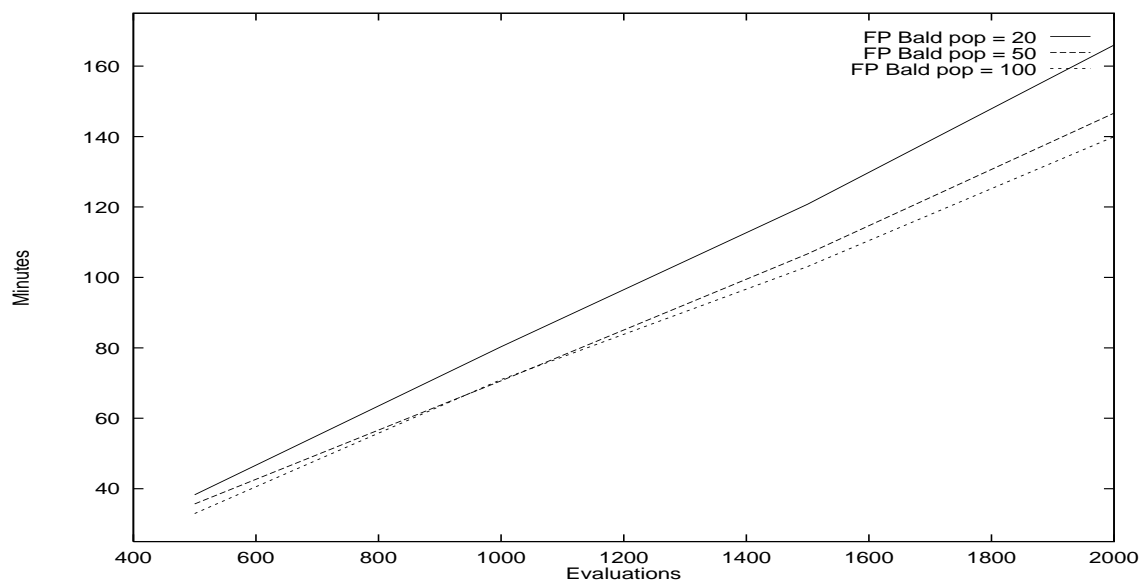
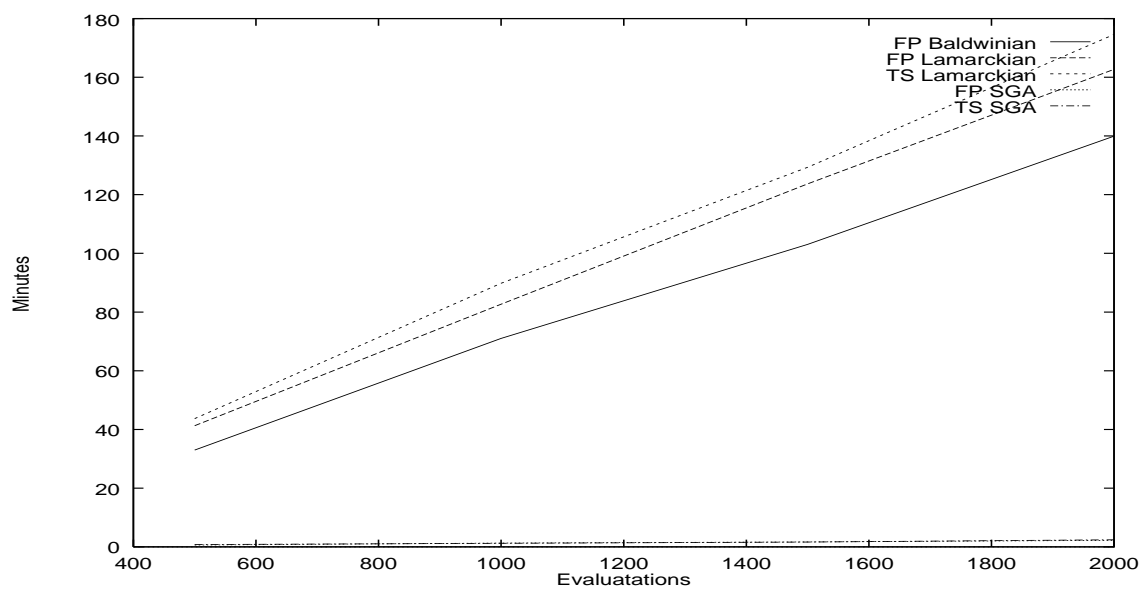
$$x \frac{x}{pop_size_{small}} > \frac{x}{pop_size_{large}}$$

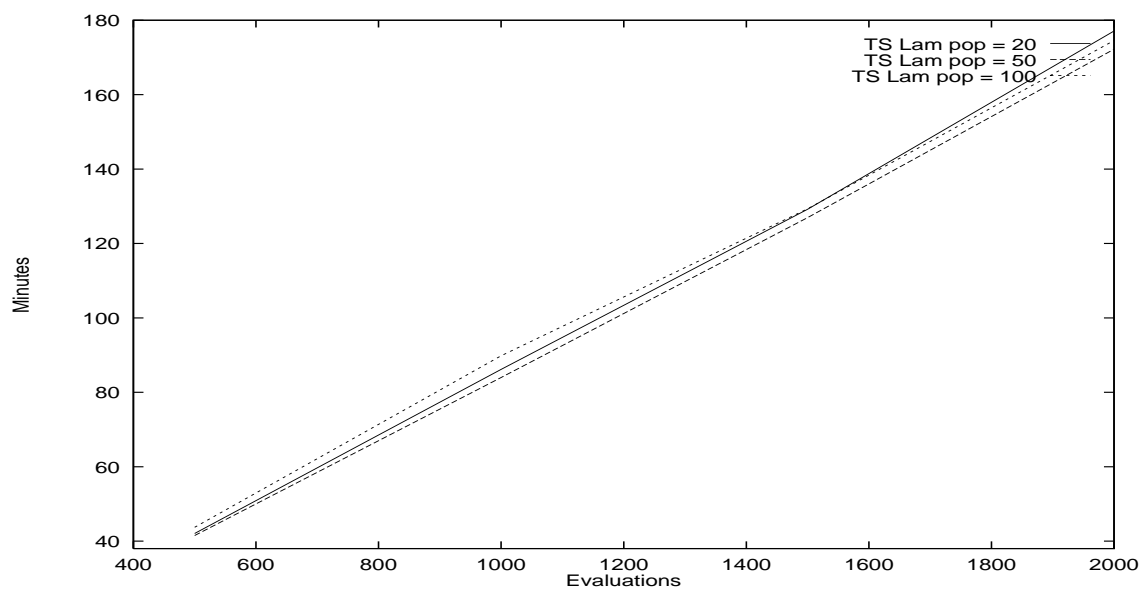
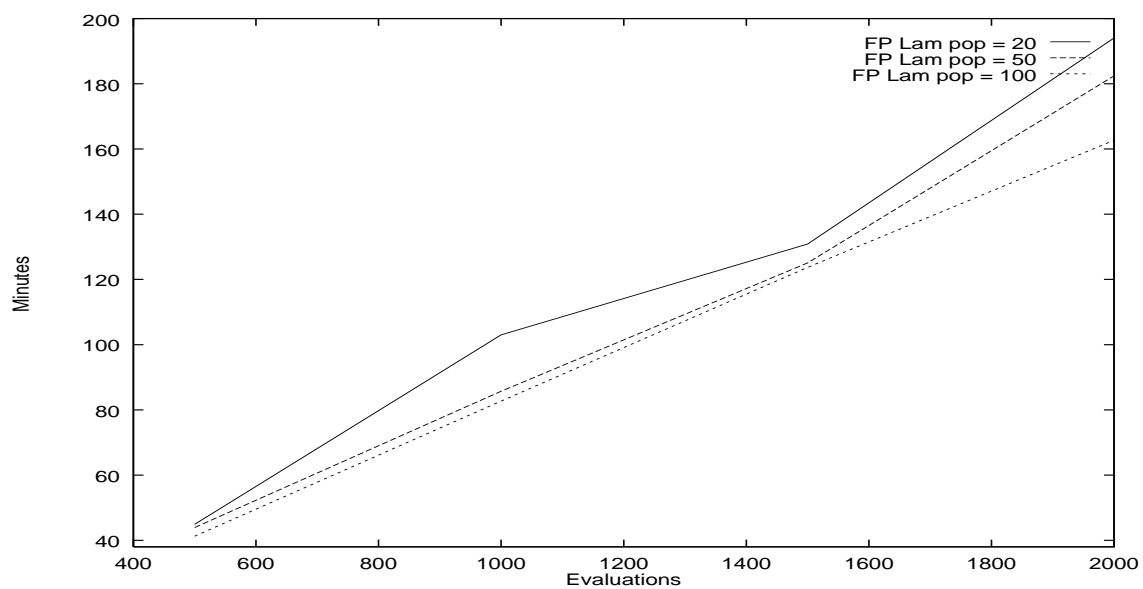


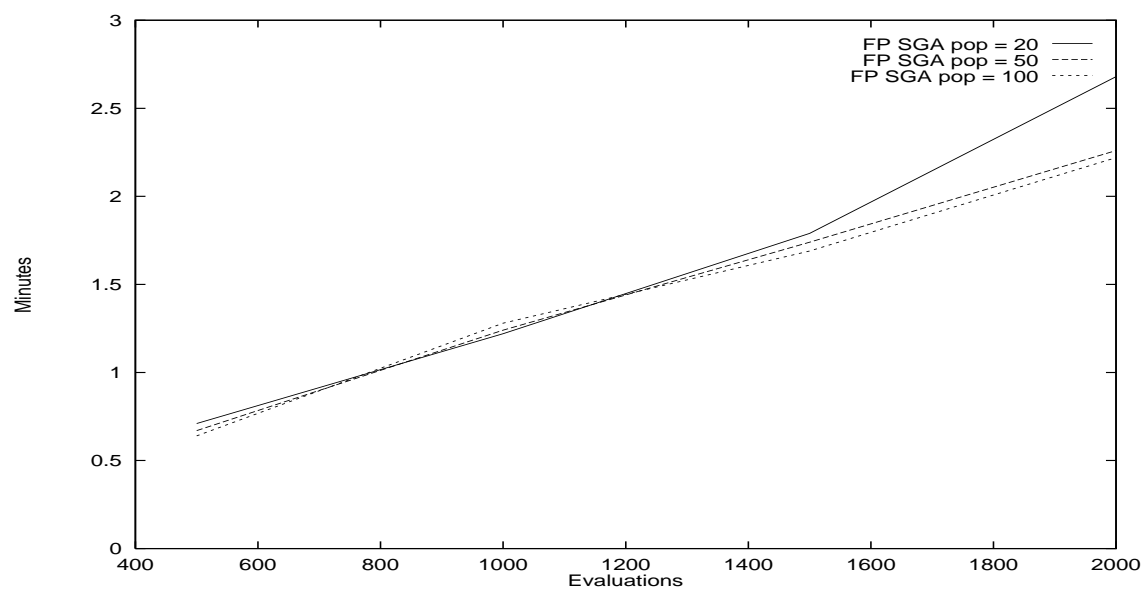
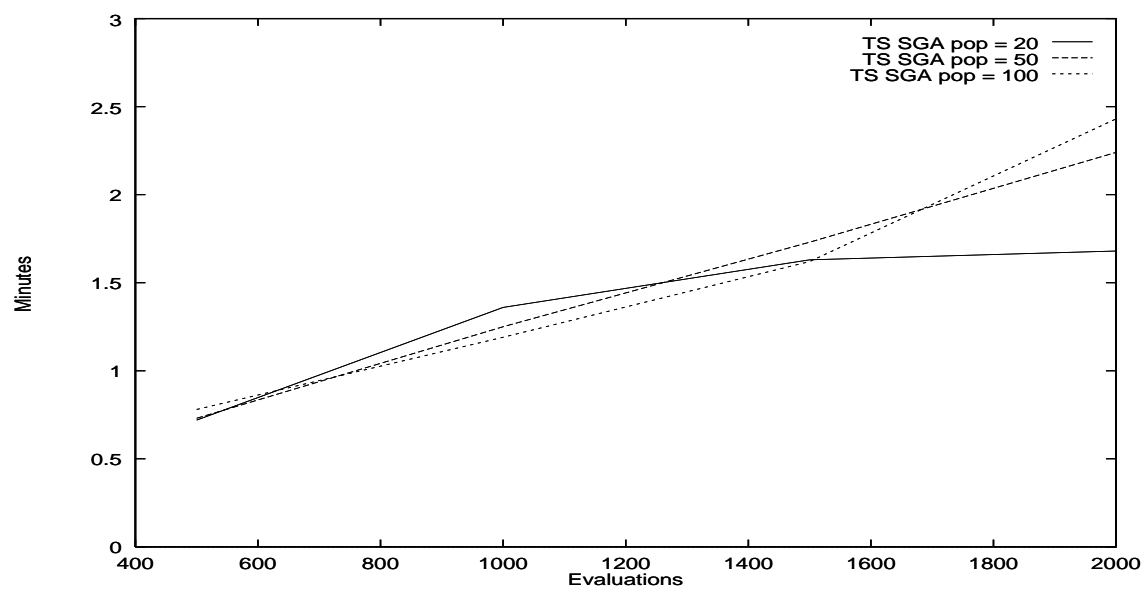


5.1.2 Efficiency Analysis, Serial.







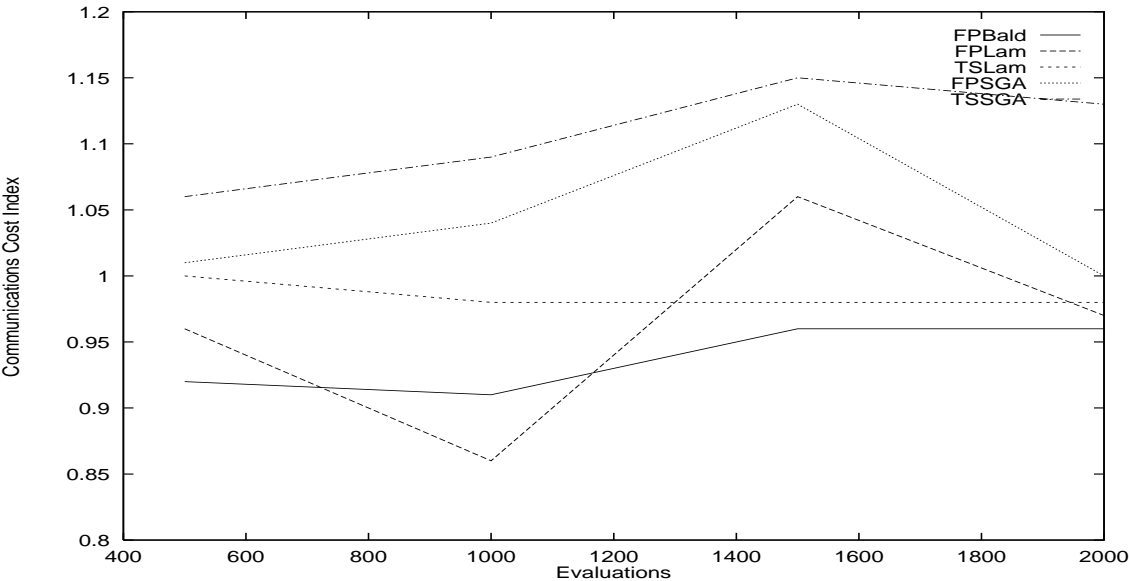


5.1.3 Efficiency Analysis, Parallel.

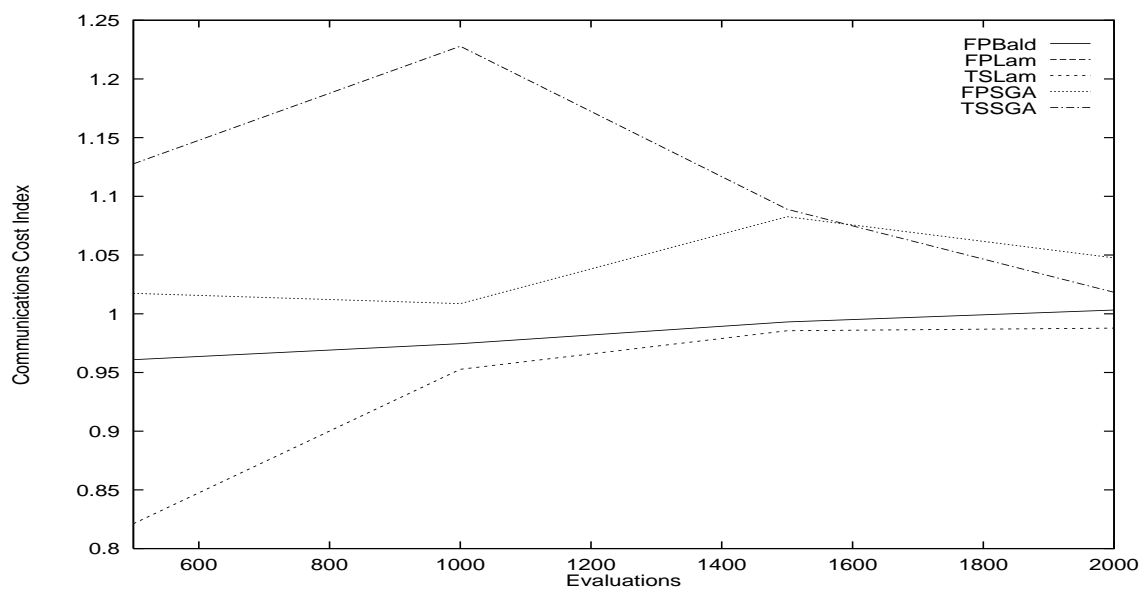
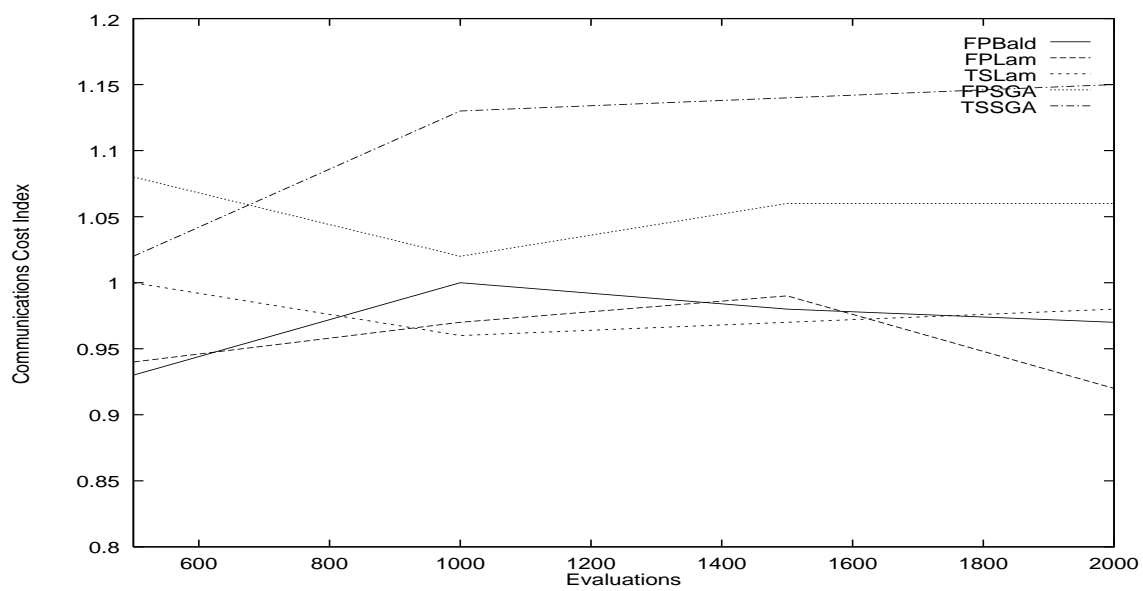
$$C_{communications} = \frac{T_{P_2} - T_S}{T_S}$$

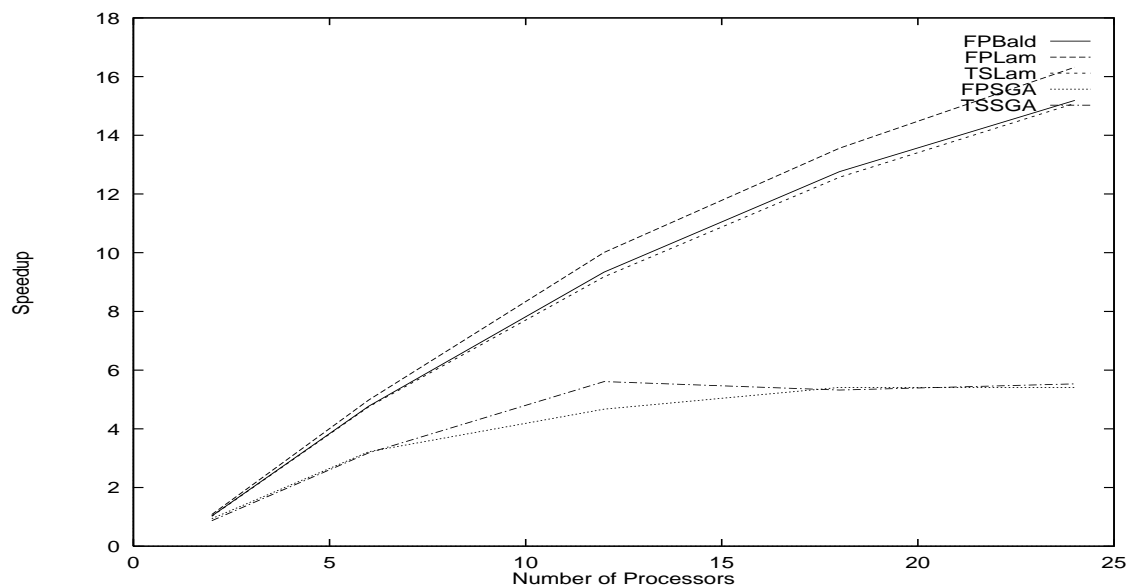
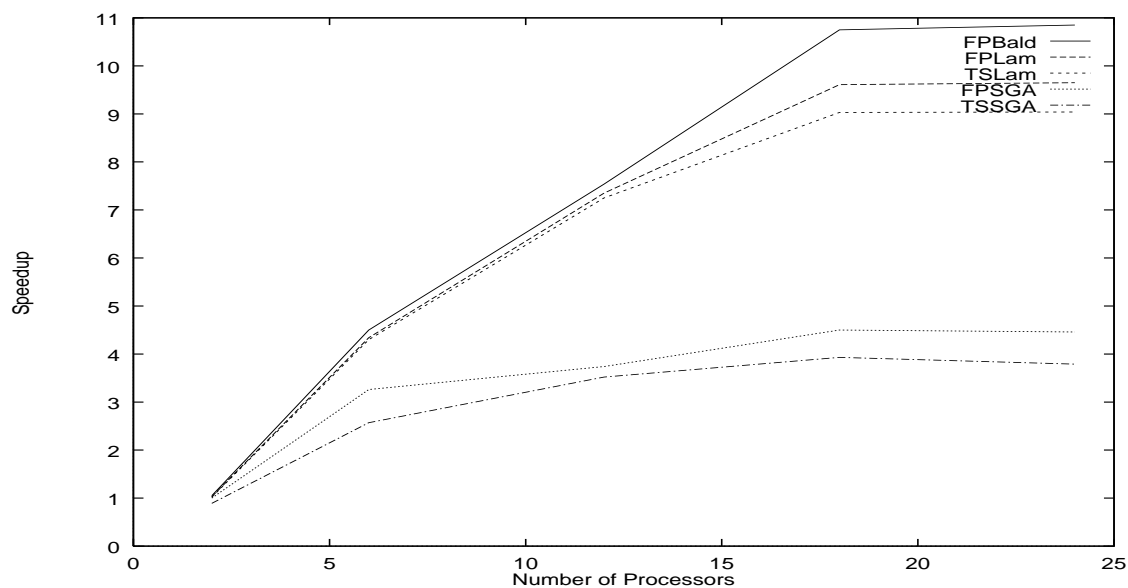
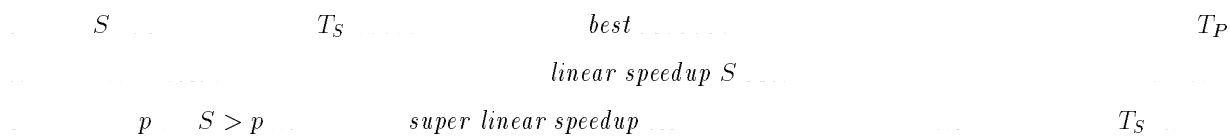
$$C_{communications} = \frac{T_{P_2}}{T_S} - 1$$

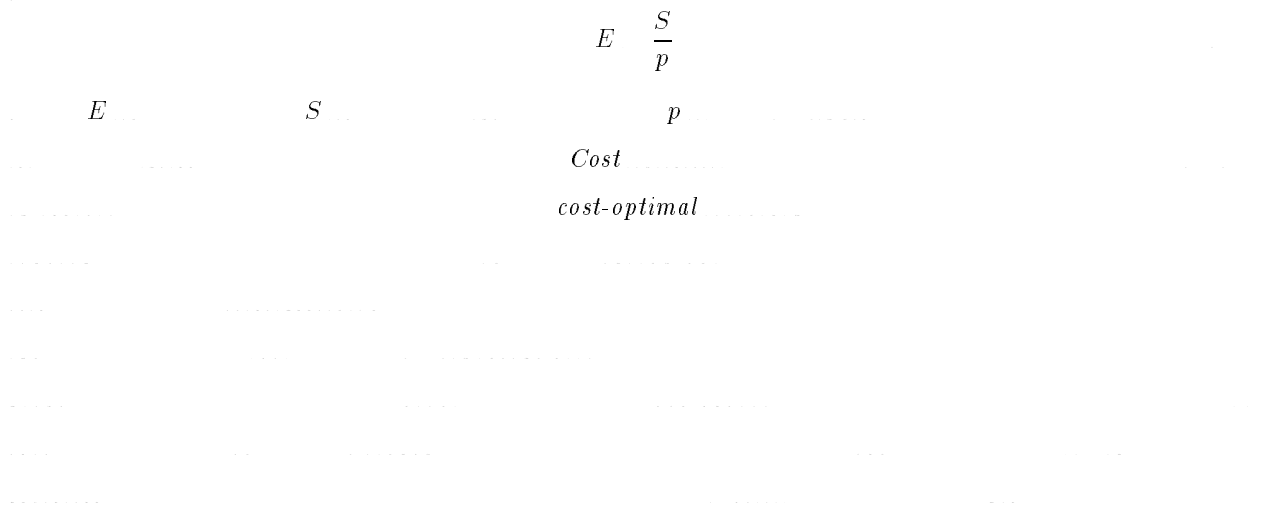
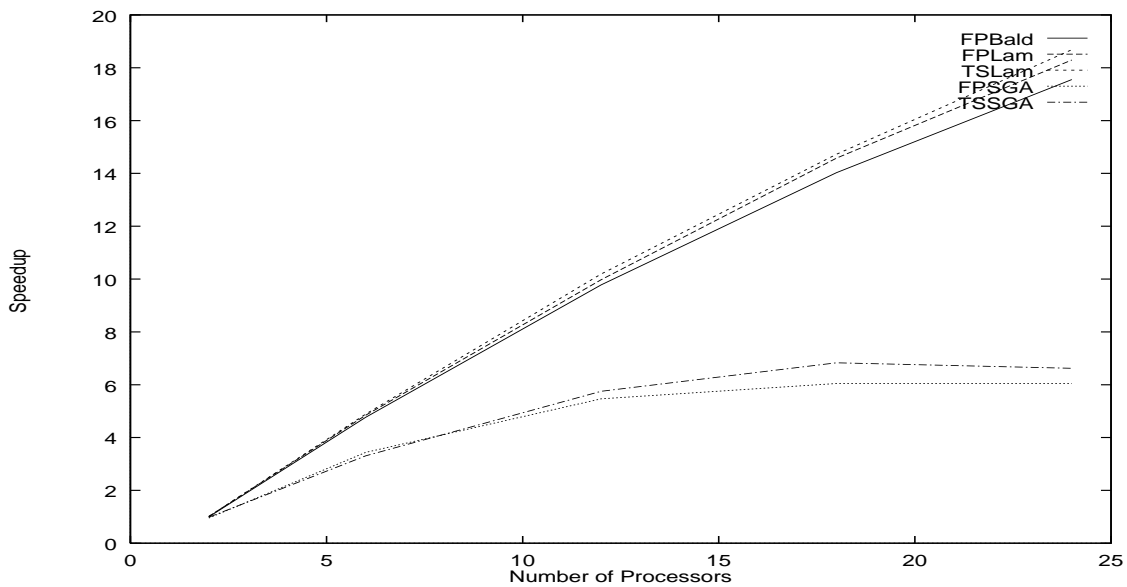
$$Communication\ Cost\ Index = \frac{T_{P_2}}{T_S}$$



$$S = \frac{T_S}{T_P}$$

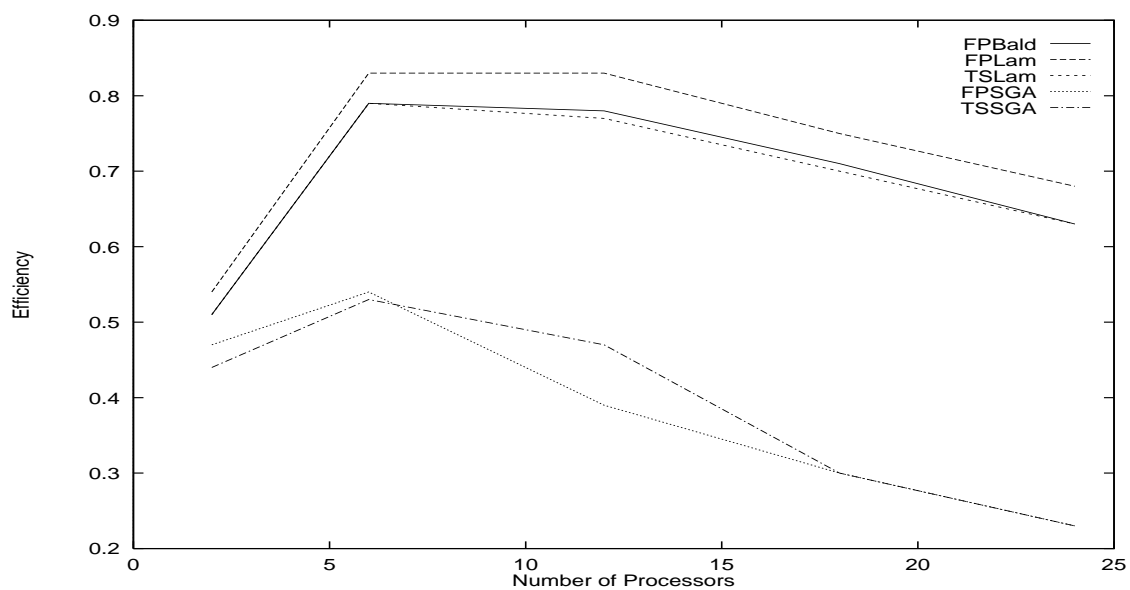
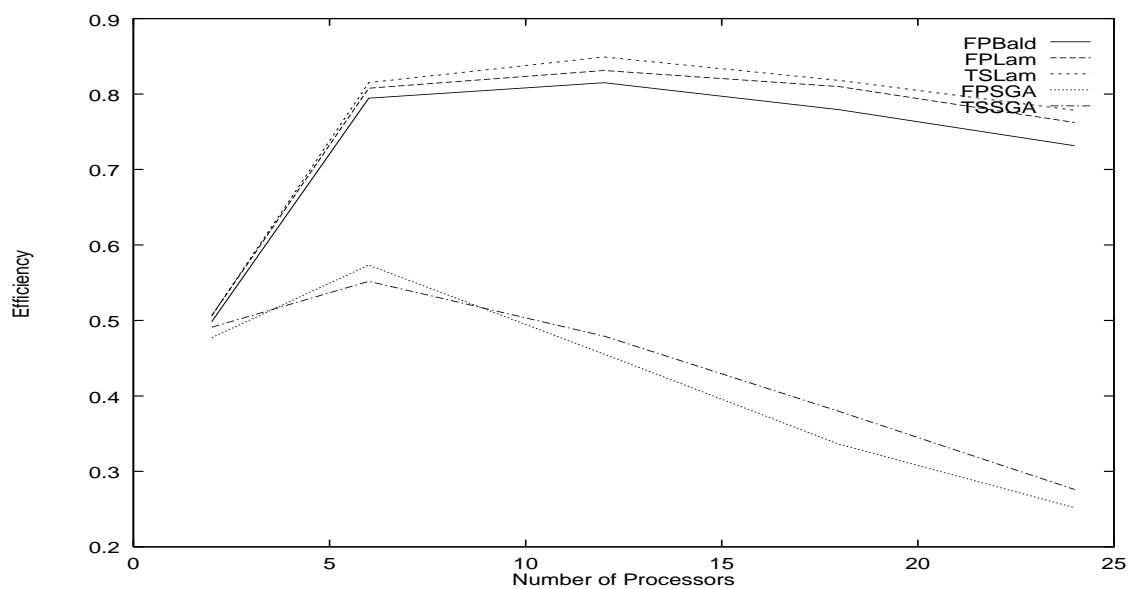


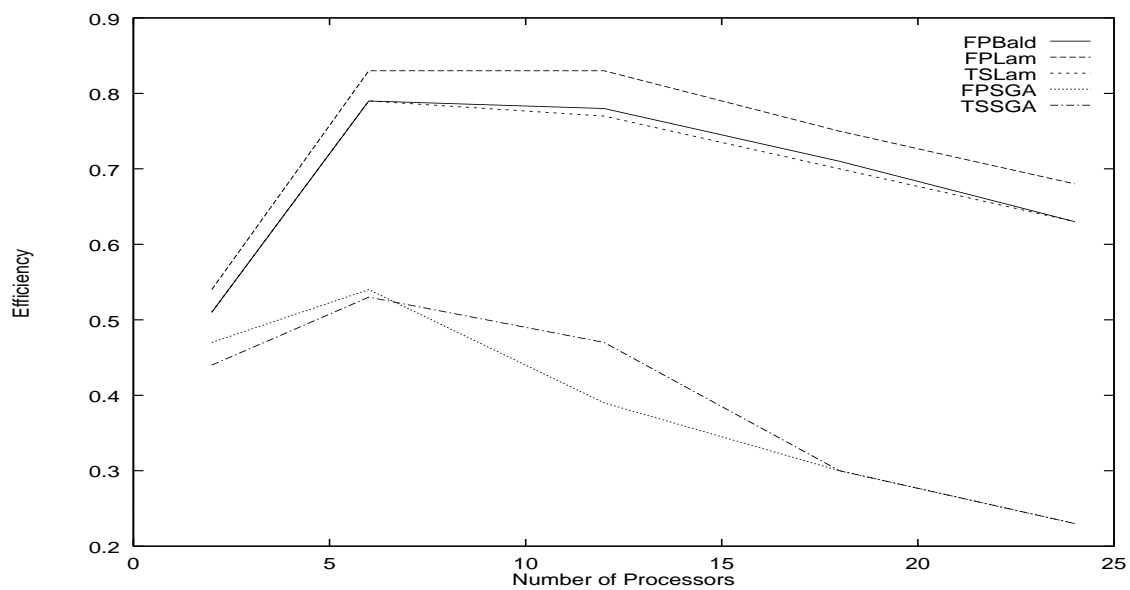




5.2 Experiment II: Preliminary REGAL Evaluation







5.2.1 [Met]-enkephalin.

\mathcal{F}

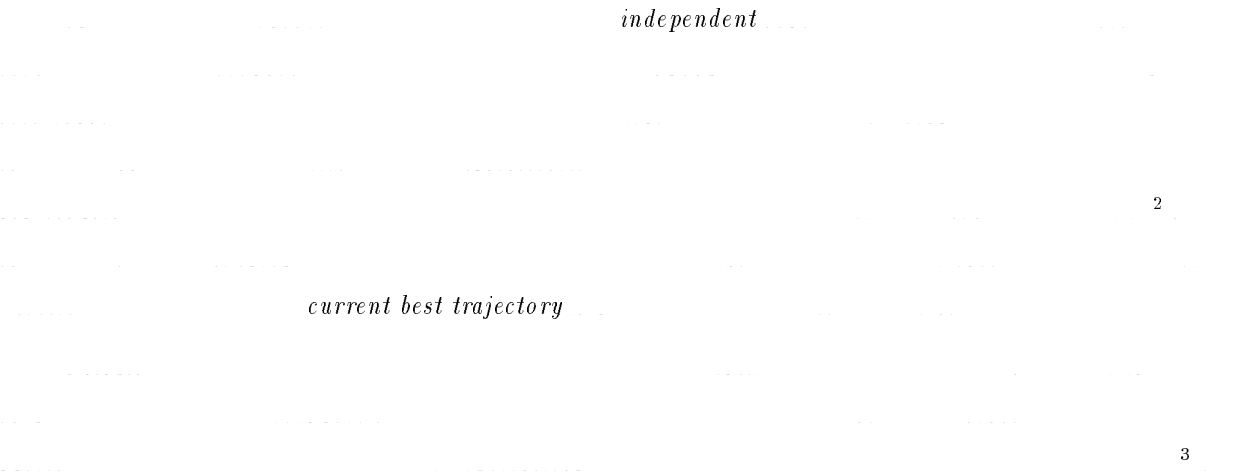
\mathcal{S}

		1	



Figure 5.3: Number of nodes in the search tree over 100 iterations. The solid line represents the number of nodes in the search tree for the independent method. The dashed line represents the number of nodes in the search tree for the current best trajectory method. The dotted line represents the number of nodes in the search tree for the independent method.

5.3 Experiment III: Analysis of Exogenous Parameters for REGAL



²Analysis of Variance, see Appendix F.1 for more details. Concern has be raised about lack of variability because a single seed set was used. The Kruskal-Wallis H Test (Appendix F.2) was used as an independent method to verify the ANOVA results. The conclusions were the same. Kruskal-Wallis results are not shown

³Hypothesis testing was not done on run times because system loading in the multi-user environment could not be controlled. They are provided for reference only. However, the large number of experiments tends to dampen out cases were the platform was heavily loaded. Thus, the data are insightful.

[illegible]

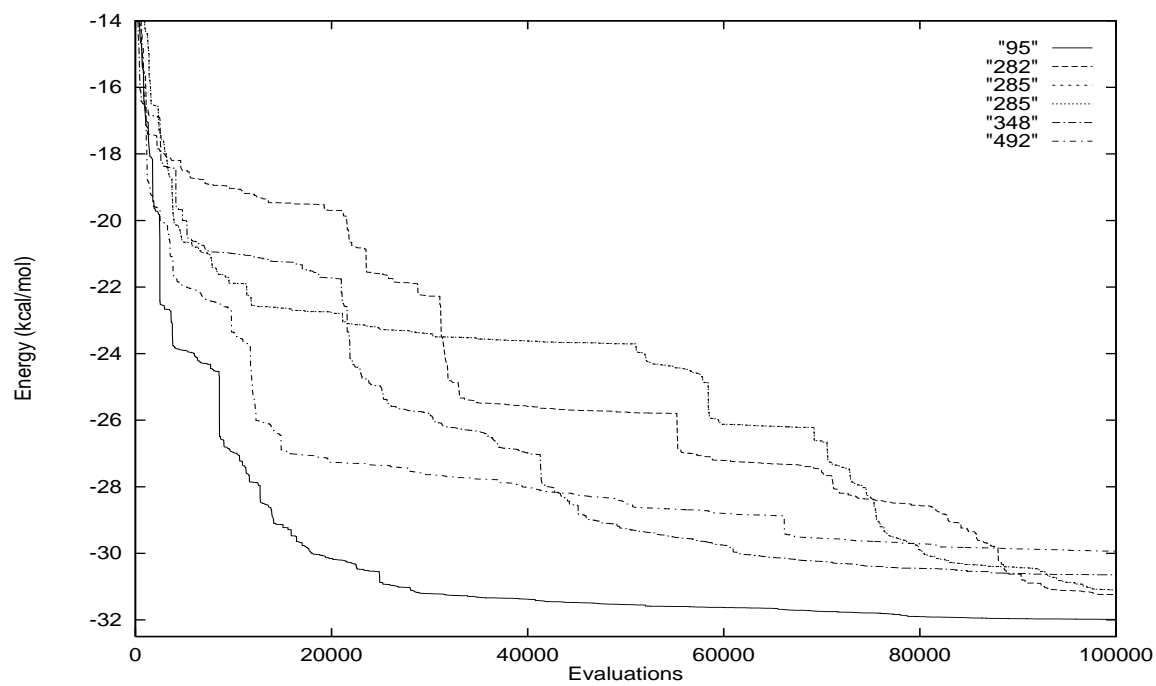
⁴This nomenclature is from the GENOCOP-III documentation. It would be more accurate to say the reference population is operated upon.



5.4 Experiment IV: Analysis of Para-REGAL



Island 3



periodicity

VI. Conclusions and Recommendation

Grand Challenge

6.1 Initiative I: PHGA

6.2 Initiative II: REGAL

trans

1

trans

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¹Actual implementation is out of scope for this investigation because research is required into appropriate control metrics.

²Limit the dihedral angle's range to a lower bound greater than $-\pi$ and an upper bound less than π .

fundamental theorem of genetic algorithms

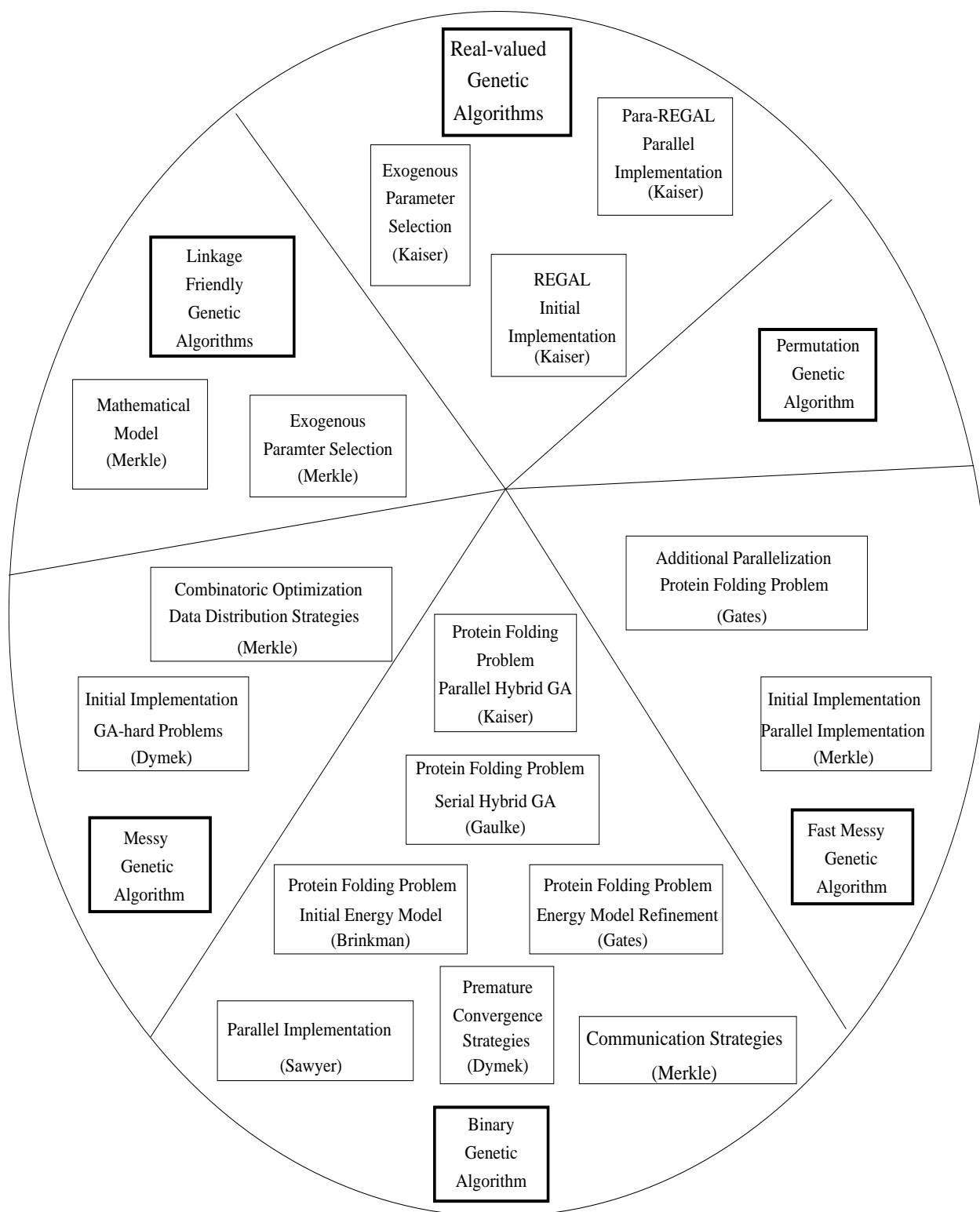
6.3 Initiative III: Examination of Exogenous Parameters

6.4 Initiative IV: Para-REGAL

molten globular

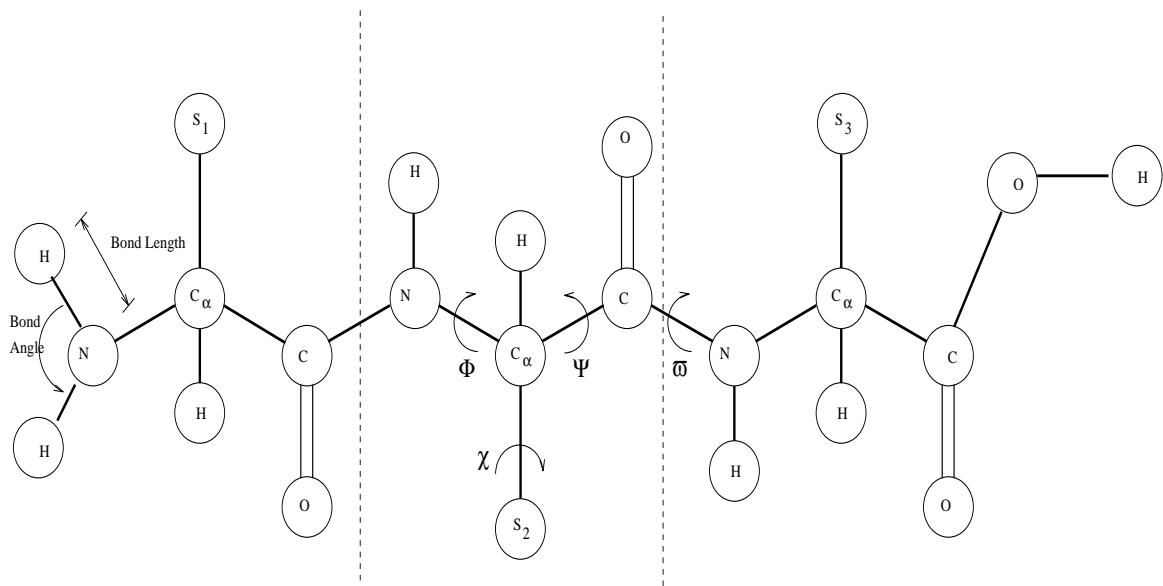
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α



6.5 Recommendations

6.6 Summary



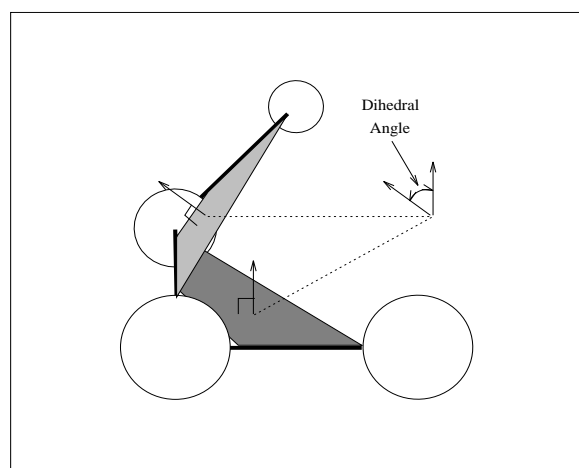
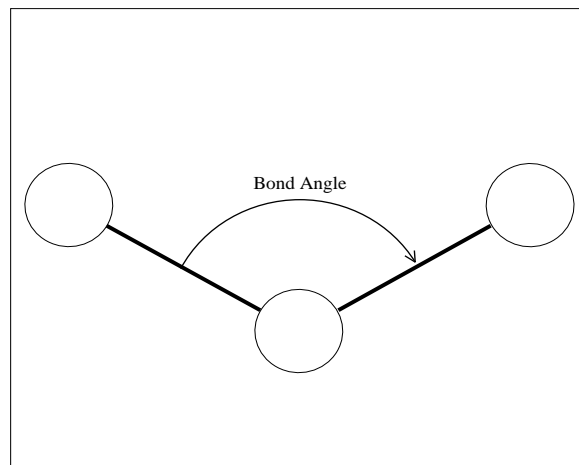
β-sheets *secondary structure* *α*-helices

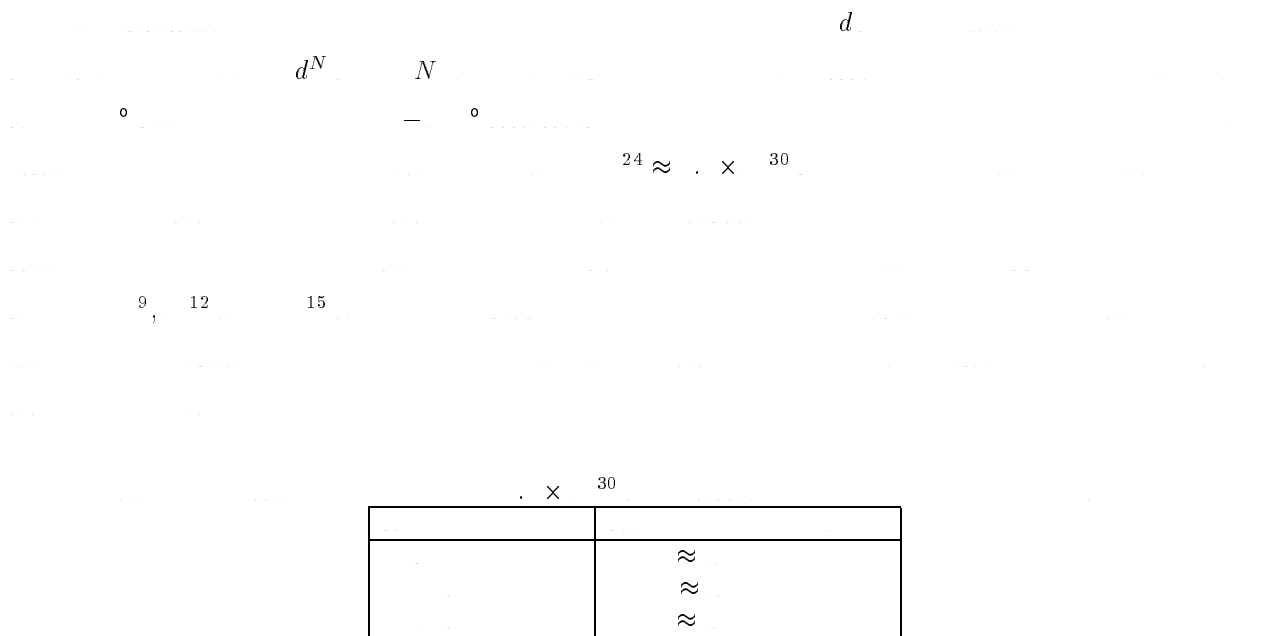
ϕ ψ

ϕ, ψ

ϕ, ψ		
	ϕ	ψ
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α		
10		
β		
β		

tertiary structure *conformation*





A.2 Experimental Tertiary Structure Determination

A.3 Tertiary Structure Prediction (PFP)



$$\begin{aligned}
 E &= \sum_{(i,j) \in \mathcal{B}} K_{r_{ij}} \left(r_{ij} - r_{eq} \right)^2 \\
 &+ \sum_{(i,j,k) \in \mathcal{A}} K_{\Theta_{ijk}} \left(ijk - r_{eq} \right)^2 \\
 &+ \sum_{(i,j,k,l) \in \mathcal{D}} K_{\Phi_{ijkl}} \left(n_{ijkl} - ijk - \gamma_{ijkl} \right) \\
 &+ \sum_{(i,j) \in \mathcal{N}} \left(\frac{A_{ij}}{r_{ij}} \right)^{12} - \left(\frac{B_{ij}}{r_{ij}} \right)^6 - \frac{q_i q_j}{\pi \varepsilon r_{ij}} \\
 &- \sum_{(i,j) \in \mathcal{N}'} \left(\frac{A_{ij}}{r_{ij}} \right)^{12} - \left(\frac{B_{ij}}{r_{ij}} \right)^6 - \frac{q_i q_j}{\pi \varepsilon r_{ij}}
 \end{aligned}$$

- \mathcal{B}
- \mathcal{A}
- \mathcal{D}
- \mathcal{N}
- \mathcal{N}'
- r_{ij} i j
- ijk $i, j,$ k
- $ijkl$ $i, j, k,$ l
- q_i i
- $K_{r_{ij}}$ r_{eq} $K_{\Theta_{ijk}}$ r_{eq} $K_{\Phi_{ijkl}}$ γ_{ijkl} A_{ij} B_{ij} ε

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Simplification

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Lattice

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Appendix B. Background on Genetic Algorithms

$$\mathbf{x}^{t+1} = \mathbf{x}^t + N(\mathbf{0}, \sigma)$$

B.2 Origins of Genetic Algorithms

1

“Adaptation in Natural and Artificial Systems”

Schema Theorem Fundamental Theorem of Genetic Algorithms

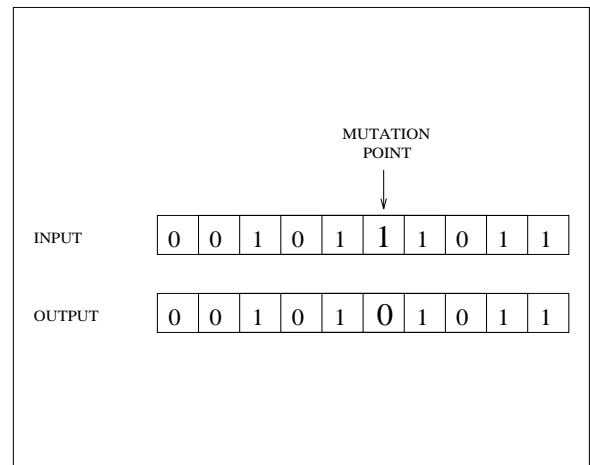
B.3 Simple Genetic Algorithm (SGA)

string chromosome genes

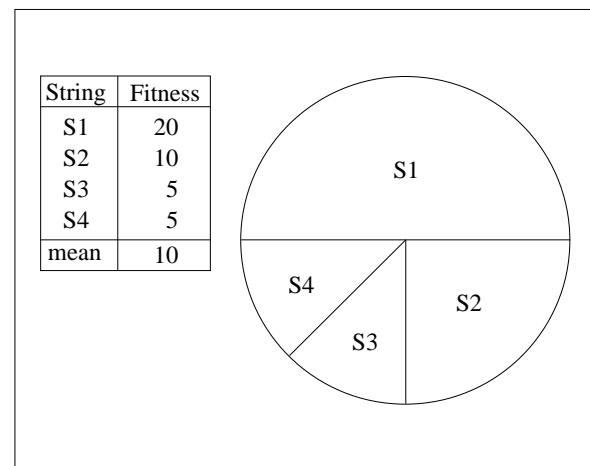
locus allele

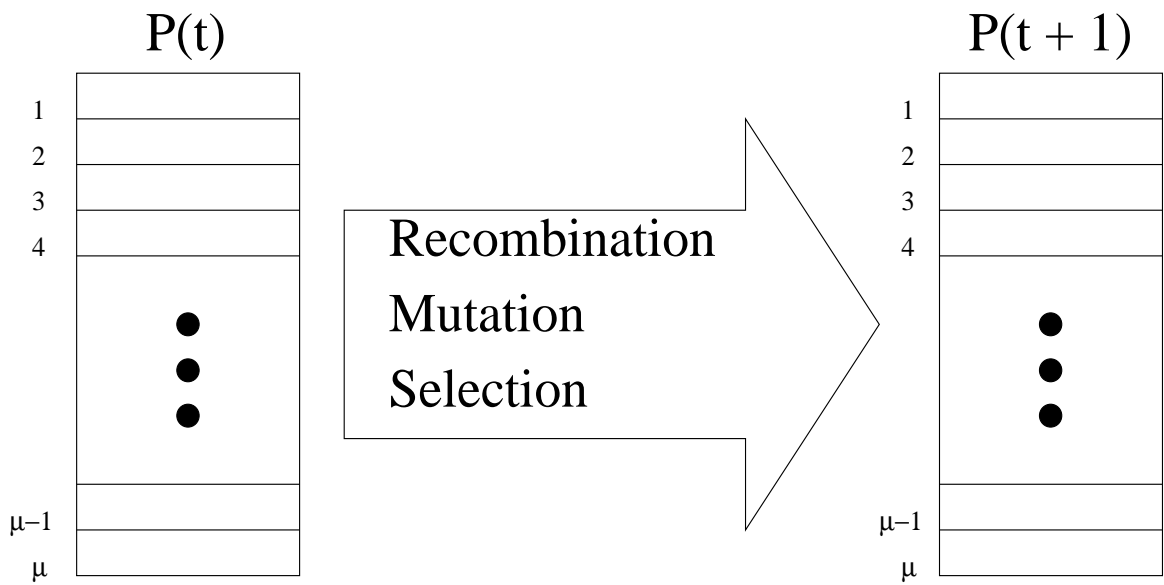
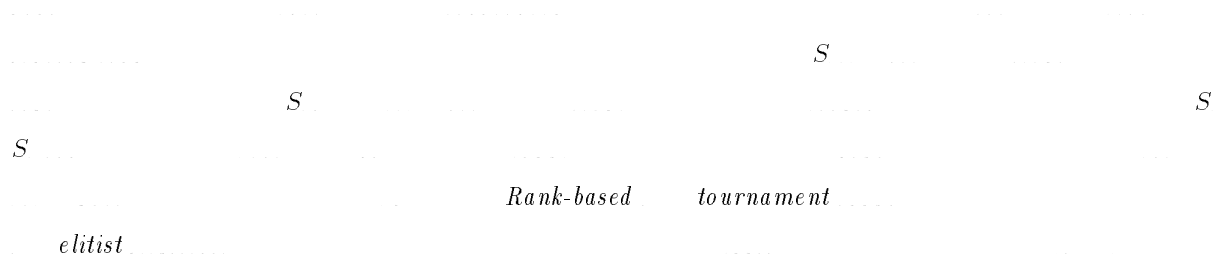
¹The term **phenotype** refers to the traits expressed by an individual, in this case the value returned by a function. Contrast this with **genotype** which refers to the traits that define the individual, for example the parameters of the function

bitwise mutation



roulette-wheel





care { } don't
 { } defining length δH order
 $o H$ H
 $\delta * - o * \delta * * -$
 $o * *$

B.4 Messy Genetic Algorithm (mGA)

B.4.1 Messy Genetic Algorithm Operators.

thresholding

cut-and-splice

cut splice

primordial juxtapositional partially enumerative initialization

(PEI)

k l
 k

B.4.3 Mathematical Theory of How (Why) Messy GAs Work.

$$\text{normalized expected defining length} = \frac{\langle \delta \rangle}{l+1} = k$$

$$\frac{\langle \delta \rangle}{l} = \frac{k -}{k}$$

B.4.3.1 Complexity Analysis.

$$\mathcal{O}(l^k)$$

$$\mathcal{O}(l - l)$$

n much

B.5 Fast Messy Genetic Algorithm (fmGA)

B.5.1 Fast Messy Genetic Algorithm Operators.

probabilistically complete initialization

B.5.3.1 Complexity Analysis.

$$\mathcal{O}(l - l)$$

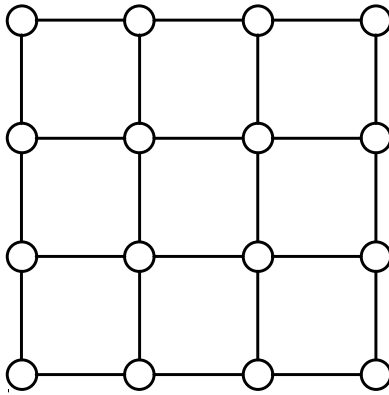
$$\mathcal{O}(l - l) = \mathcal{O}(l^k)$$

$$\mathcal{O}(nl)$$

Appendix C. Background on Parallel Computing

C.1 Parallel Architectures

	<i>node</i>
	<i>granularity</i>
	<i>fine grained</i>
	<i>coarse grained</i>



×

shared memory

distributed memory

message passing

interconnection topology network

2-D mesh

×

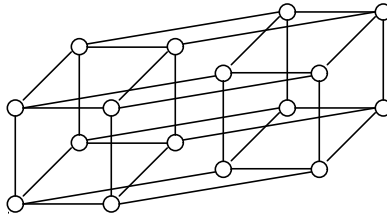
hypercube

dimension N

N

N

N



C.2 Parallel Algorithms.

Consider the following algorithm for computing the sum of the elements of an array A of size n .

```

1  sum ← 0
2  for i ← 1 to n
3    sum ← sum + A[i]
4  return sum

```

Suppose that the array A is partitioned into p blocks, each of size n/p .

Consider the following algorithm for computing the sum of the elements of an array A of size n .

```

1  sum ← 0
2  for i ← 1 to n/p
3    sum ← sum + A[i]
4  return sum

```

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Consider the following algorithm for computing the sum of the elements of an array A of size n .

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Suppose that the array A is partitioned into p blocks, each of size n/p .

Consider the following algorithm for computing the sum of the elements of an array A of size n .

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2  for i ← 1 to n/p
3    sum ← sum + A[i]
4  return sum

```

Suppose that the array A is partitioned into p blocks, each of size n/p .

- The array A is partitioned into p blocks, each of size n/p .
- The array A is partitioned into p blocks, each of size n/p .
- The array A is partitioned into p blocks, each of size n/p .

●

●

```

.....
.....
.....
in      in.something
something ..... in
.....
.....
.....
in.something .....
.....
.....
> .....
/dev/null
.....
..... psga_param
psga_default ..... psqa_param .....
..... psqa_default .....
in
mpirun .....
-np
-sz
-
-
in.2.10.24 .....
& ..... >
/dev/null
```

Experiments = 1
Total Trials = 500
Population Size = 20
Structure Length = 240
Crossover Rate = 0.65
Mutation Rate = 0.005
Generation Gap = 1.0
Scaling Window = 1
Report Interval = 1
Structures Saved = 1
Max Gens w/o Eval = 10
Dump Interval = 0
Dumps Saved = 0
Options = ycel
Number of Peaks = 1.0
Minimization Prob = 1.0
Replacement Prob = 1.0
Random Seed = 987654321
Rank Min = 1.5

Appendix E. Genocop-III

E.1 Algorithm

Procedure Genocop III

begin

$t \leftarrow t$

$P_s t$

$P_r t$

$P_s t$

$P_r t$

while not do

begin

$t \leftarrow t$

$P_s t$ $P_s t -$

$P_s t$

$P_s t$

if $t \bmod k$ then

begin

$P_r t$

$P_r t$ $P_r t -$

$P_r t$

end

end

end

P_s

algorithm domain

genocop.h

- ¹ Here the phrase “Domain Constraints” is used in a context more limited than normally used in compute science, specifically, the allowable range of specified variables. If a domain constraint is not defined for a variable, it defaults to the architecture dependent range for R

-		$N \cup$
-		$N \cup$
-		$N \cup$
-		$N \cup$
-		$N \cup$
- -		N
- - -		N
-		N
- -		N
- - -		N
- -		N
- -		N
- - -		
- -		
-		
- - -		
- -		
-		
-		N
		R
		$\leq iSeed \leq$
		$\leq iSeed \leq$
-		

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E.3 Operators

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$$\begin{aligned} E.3.1 \quad \text{Whole arithmetical crossover.} \quad & \vec{z} = a\vec{x} + (1-a)\vec{y} \\ \vec{x} \quad \vec{y} \quad & \leq a \leq \end{aligned}$$

$$\begin{aligned} E.3.2 \quad \text{Simple arithmetical crossover.} \quad & \\ & \text{single point crossover} \end{aligned}$$

$$\begin{aligned} E.3.3 \quad \text{Whole uniform mutation.} \quad & \vec{x} = x_1, \dots, x_k, \dots, x_n \\ \vec{x}' = x_1, \dots, x'_k, \dots, x_n \quad & x'_k \\ & k \end{aligned}$$

$$E.3.4 \quad \text{Boundary mutation.} \quad x'_k \quad \text{left } k \quad \text{right } k$$

$$E.3.5 \quad \text{Non-uniform mutation.}$$

$$\begin{aligned} x_k^{t+1} &= \begin{cases} x_k^t + \Delta(t, r, k) - x_k & \text{if } r < \frac{t}{T} \\ x_k^t - \Delta(t, x_k - l, k) & \text{if } r \geq \frac{t}{T} \end{cases} \\ & \quad k = 1, \dots, n \\ \Delta(t, y) &= \begin{cases} t & \text{if } y < \frac{t}{T} \\ t & \text{if } y \geq \frac{t}{T} \end{cases} \\ & \quad t = \frac{t}{T} \\ & \quad t = \frac{t}{T} \\ & \quad \Delta(t, y) = y \cdot r \cdot \left(-\frac{t}{T} \right)^b, \\ & \quad r = \dots T \dots b \end{aligned}$$

E.3.6 Whole non-uniform mutation.

E.3.7 Heuristic crossover.

heuristic crossover

$$\vec{z} = \begin{cases} \vec{x} & \text{if } f(\vec{x}) \leq f(\vec{y}) \\ \vec{y} & \text{if } f(\vec{x}) > f(\vec{y}) \end{cases}$$

$$\vec{z} = r \cdot (\vec{x} - \vec{y}) + \vec{x}$$

$$r = \frac{f(\vec{x}) - f(\vec{y})}{f(\vec{x}) - f(\vec{y})}$$

E.3.8 Gaussian mutation.

Gaussian mutation

$$\vec{x} = \langle x_1, \dots, x_n \rangle$$

$$\vec{x}^{t+1} = \vec{x}^t + N(0, \sigma)$$

$$N(0, \sigma)$$

$$\vec{\sigma}$$

E.3.9 Pool recombination operator.

E.3.10 Scatter search operator.

$$\vec{b} = \vec{w} + \vec{c}$$

$$\vec{c} = \vec{x}_i / k - \vec{w}$$

$$\vec{y} = \vec{c} - \vec{w}$$

.....
.....
.....

orgy

Appendix F. Statistical Methods

F.1 Analysis of Variance (ANOVA)

F.1.1 Single Factor Factorial Design a

. n a

.

$y_{ij} = \mu + \tau_i + \epsilon_{ij}$ $i = 1, \dots, a$
 $j = 1, \dots, n$

y_{ij} i, j μ overall mean τ_i
 i treatment effect ϵ_{ij}

.

. σ^2

.

. fixed
effects model only

.

. random samples random effects model τ_i

.

. τ_i

. a n

. N $\sum_{i=1}^a n_i$

analysis of variance

$$SS_T = \sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_{..})^2$$

$$SS_T =$$

.....

$$SS_T = SS_{Treatments} + SS_E$$

$$SS_{Treatments} = \sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_{i.})^2 \qquad SS_E = \sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_{i.})^2$$

..... no difference $H_0: \mu_1 = \mu_2 = \dots = \mu_a$

$$H_0: \tau_1 = \tau_2 = \dots = \tau_a$$

.....

$$F_0 = \frac{SS_{Treatments}/(a-1)}{SS_E/(N-a)} = \frac{MS_{Treatments}}{MS_E}$$

$$F > F_{\alpha, a-1, N-a} \qquad H_0$$

$$F_0 > F_{\alpha, a-1, N-a}$$

$$F_0 = \frac{SS_{Treatments}/(a-1)}{SS_E/(N-a)}$$

.....

$$SS_T = \sum_{i=1}^a \sum_{j=1}^n y_{ij}^2 - \frac{y_{i.}^2}{n}$$

$$SS_{Treatments} = \sum_{i=1}^a \frac{y_{i.}^2}{n} - \frac{y^2}{N}$$

$$SS_E = SS_T - SS_{Treatments}$$

..... y

				F_0
	$SS_{Treatments}$	$a-1$	$MS_{Treatments}$	$F_0 = \frac{MS_{Treatments}}{MS_E}$
	SS_E	$N-a$	MS_E	
	SS_T	$N-1$		

$$y_{i.} = \sum_{j=1}^n y_{ij}, \quad y_{i.} = y_{i.}/n \quad i = 1, \dots, a$$

$$y_{..} = \sum_{i=1}^a \sum_{j=1}^b y_{ij}, \quad y_{..} = y_{..}/N$$

F.1.2 Two Factor Factorial Design

$$y_{ijk} = \mu + \tau_i + \beta_j + \tau\beta_{ij} + \epsilon_{ijk} \quad \begin{matrix} i = 1, \dots, a \\ j = 1, \dots, b \\ k = 1, \dots, n \end{matrix}$$

$$\begin{matrix} \mu & \tau_i & i & A & \beta_j \\ j & B & \tau\beta_{ij} & \tau_i & \beta_j & \epsilon_{ijk} \\ & A & & & & \end{matrix}$$

$$\begin{aligned} H_0 &: \tau_1 = \tau_2 = \dots = \tau_a \\ H_1 &: \exists \tau_i \neq 0 \end{aligned}$$

$$\begin{aligned} H_0 &: \beta_1 = \beta_2 = \dots = \beta_b \\ H_1 &: \exists \beta_i \neq 0 \end{aligned}$$

$$\begin{aligned} H_0 &: \tau\beta_{ij} = 0 \quad \forall i, j \\ H_1 &: \exists \tau_i \neq 0 \end{aligned}$$

$$H_0$$

				F_0
	SS_A	$a - 1$	$MS_A = \frac{SS_A}{a-1}$	$F_0 = \frac{MS_A}{MS_E}$
	SS_B	$b - 1$	$MS_B = \frac{SS_B}{b-1}$	$F_0 = \frac{MS_B}{MS_E}$
	SS_{AB}	$(a-1)(b-1)$	$MS_{AB} = \frac{SS_{AB}}{(a-1)(b-1)}$	$F_0 = \frac{MS_{AB}}{MS_E}$
	SS_E	$ab(n-1)$	$MS_E = \frac{SS_E}{ab(n-1)}$	
	SS_T	$abn - 1$		

Suppose we have k independent samples from k populations. We wish to test the null hypothesis

H_0 : the samples are from identical populations
against the alternative hypothesis

H_1 : the populations are not identical
at the α level of significance.

1. Compute h . Calculate

$$h = \frac{12}{n(n+1)} \sum_{i=1}^k \frac{R_i^2}{n_i} - 3(n+1)$$

2. Accept or reject H_0 . If $h > \chi_{k-1, \alpha}^2$, reject H_0 ; otherwise accept H_0 .

(1)

Probability, Statistics, and Queueing Theory: With Computer Science Applications

Journal of Global Optimization

Evolutionary Algorithms in Theory and Practice

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